7. Optimal Rod Programming and Loading Pattern in a Multiregion Reactor, *Hiroshi Motoda, Toshio Kawai, Takashi Kiguchi (Hitachi-Japan)*

A theory of optimal control rod programming has been developed for a two-region reactor model and several important results have been obtained.[1,2] The significant characteristic of the burnup optimization problem is that the optimal terminal state (OTS) can be uniquely determined depending on the initial state and the control freedom.[3] Therefore, the problem is focused on investigating the nature of the OTS and synthesizing the optimal control rod programming during one refueling interval.

To clarify this point and find the more realistic control rod programming, the method of approximation programming[4] is applied to a one-dimensional multiregion slab reactor.

The problem is to minimize Eq. (1), satisfying Eqs. (2) through (5).

$$J = \int_0^{H/2} k_0(x)dx \tag{1}$$

$$M^2 \frac{\partial^2\phi(x,t)}{\partial x^2} + [k_0(x) - 1 - \alpha(x)e(x,t) - u(x,t)]\,\phi(x,t) = 0 \quad , \quad 0 \leq x \leq H/2 \quad , \quad 0 \leq t \leq t_f \tag{2}$$

$$\frac{\partial e(x,t)}{\partial t} = \phi(x,t) \quad , \quad 0 \leq x \leq H/2 \quad , \quad 0 \leq t \leq t_f \tag{3}$$

$$\int_0^{H/2} \phi(x,t)dx = \frac{H}{2} \quad , \quad 0 \leq t \leq t_f \tag{4}$$

$$\left.\begin{array}{ll} 0 \leq \phi(x,t) \leq f & , \; 0 \leq e(x,t) \leq E \\ 0 \leq u(x,t) \leq U & , \; k_{min} \leq k_0(x) \leq k_{max} \end{array}\right\} \; . \qquad 0 \leq x \leq H/2 \quad , \quad 0 \leq t \leq t_f \tag{5}$$

This problem is equivalent to maximizing the average burnup of a given initial loading pattern if the relative spatial distribution of $k_0(x)$ is fixed.

Original equations and constraints which are continuous in space x and time t are discretized by central difference and forward difference approximations, resulting in a set of nonlinear equations of neutron flux $\phi_{n,m}$, burnup $e_{n,m}$, control rod density $u_{n,m}$, and initial fuel property k_{0n} at space and time mesh point $n(n = 1 \sim N)$ and $m(m = 1 \sim M)$. These equations are further linearized with respect to these variables to use linear programming repeatedly.

The results obtained to date are as follows:

1. Although the optimal terminal exposure pattern is uniquely determined for every initial fuel loading pattern, the control rod programming is not necessarily unique.

   a. It is unique when some of the rods have to remain inserted at the end-of-life (EOL).

   b. It is unique when all the rods can be fully withdrawn and the power peaking factor at EOL is below the constraint.

   c. It is not unique otherwise except for special cases.

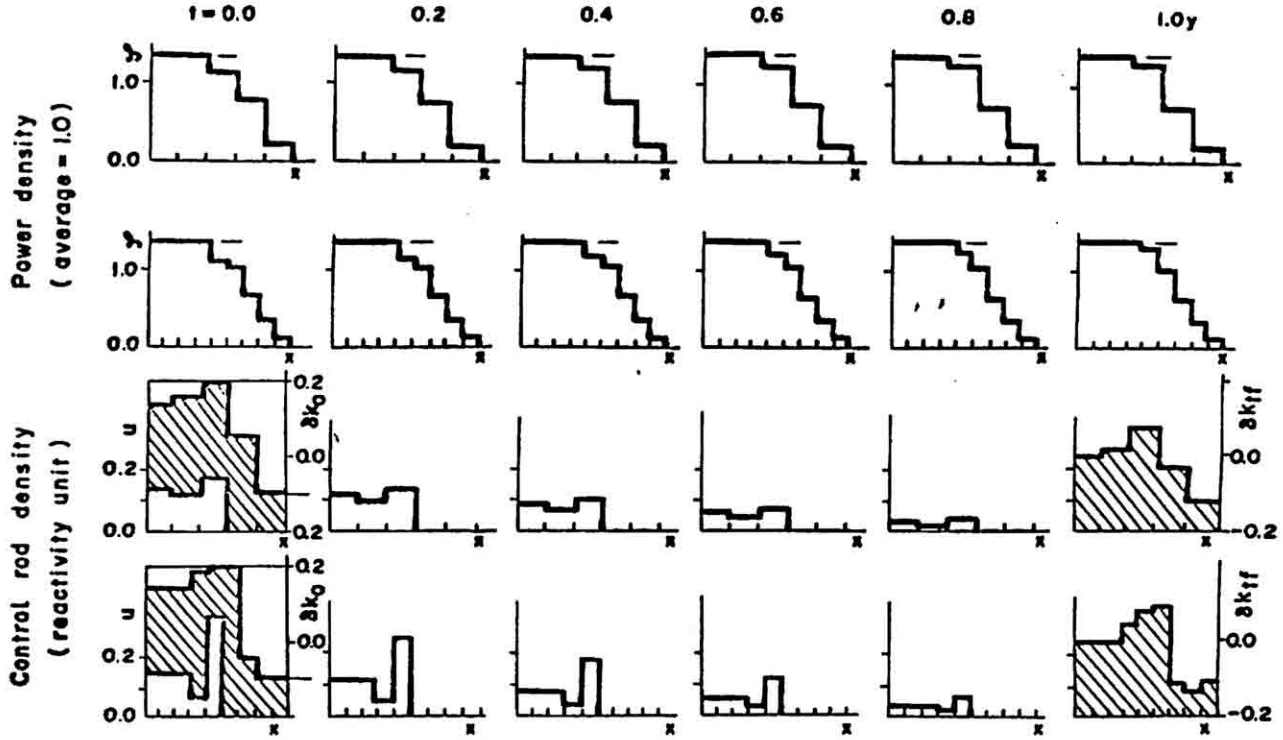

Fig. 1. Optimal loading pattern and optimal control rod programming $f = 1.4$, $U = 0.5$, $E = 2.0$, $k_{min} = 0.9$, $k_{max} = 1.2$, $t_f = 1.0$ y.

The first and the third rows are of $N = 5$, $Nu = 5$, $M = 11$, and the second and the fourth are of $N = 9$, $Nu = 9$, $M = 6$. The optimal loading pattern is a three-region bang-bang type. The nuclear property of the inner region at the beginning-of-life (BOL) is uniquely determined by the maximum allowable power peaking factor f. The power density in this region is flat and maximum, and the control rod density is also flat, giving the net nuclear property k_∞ of 1.0 during the whole operation period, and is reduced to zero at EOL. The nuclear property of the middle and the outer region at BOL is fixed at k_{max} and k_{min}, respectively. The control rod density at these regions is such as to make the power distribution as outer high as possible.

2. Good agreement is obtained between the solution of the two-region model[1,2] and the present calculation with two degrees of control freedom, $Nu = 2(Nu \leq N)$. Multiregion control results in a burnup gain about three times larger than the above results by the increased degree of control freedom.

3. The optimal loading pattern is a three-region bang-bang type under one-group approximation, which is very similar to that of the minimum critical mass problem with the constraints on power peaking and fuel property.[5] The corresponding optimal control rod programming is unique and its policy is globally "inner-high"[2] and locally "outer-high." This is shown in Fig. 1. From the physical consideration, this result is understandable.

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