## 7. On-Line Core Performance Prediction of BWR, *Yasuo Nishizawa, Takashi Kiguchi, Hiroshi Motoda (AERL-Japan)*

Recent operating experience of BWR has indicated the need to develop a tool to predict the change in power distribution in advance of control rod motion, flow rate change, and xenon transient initiation.

It is indispensable to have correct knowledge of present power distribution before beginning the predictional calculation. A method has been developed to estimate the power level and its axial distribution of a monitored channel using LPRM readings alone. An axially one-dimensional (1-D) FLARE-type[1] nuclear thermal-hydraulic calculation is performed for that channel. Channel power, and radial neutron leakage expressed in terms of "neutron reflection rate $\beta(k)$" in Eq. (1) are iteratively adjusted so the calculated TIP readings are equal to the measured LPRM readings at its locations.

$$S(k) = k_\infty(k)\Big(S(k-1)\,W^V(k-1) + S(k+1)\,W^V(k+1) + S(k)\left\{1 - 2W^V(k) - 4W^H(k)\,[1-\beta(k)]\right\}\Big) \quad (1)$$

Numerical experiments show that the maximum error in the calculated TIP reading is about 5%. This part is unnecessary when the present TIP data are available.

Primal incentive of core performance prediction is to evaluate the prompt change in linear heat rate by control rod withdrawals at reactor startup, particularly for operations at the minimum flow rate. The first portion of the prediction part is the model identification. Using the estimated monitored channel $S_l(k)$'s, 3-D FLARE-type nodal Eq. (2) is solved for $k^0_{\infty l}(k)$ by setting $\lambda = 1$. Changes of $k_\infty$ due to the control rod motions and the associated void changes are linearized: $k_{\infty l}(k) = k^0_{\infty l}(k) + \Delta k_{\infty l}(CR, v)$, and Eq. (2) is solved for the 4 monitored channels surrounding the moved control rod shown in the upper right of Fig. 1.

$$S_l(k) = \frac{k_{\infty l}(k)}{\lambda}\Big[\sum_{i=1}^{4} W^H_i(k)S_i(k) + W^V_l(k-1)S_l(k-1) + W^V_l(k+1)S_l(k+1) + W^S_l(k)S_l(k)\Big] \quad (2)$$

The small changes of the power of the shaded channels are taken into account by Eq. (3). Equations (2) and (3) form an inhomogeneous source problem.

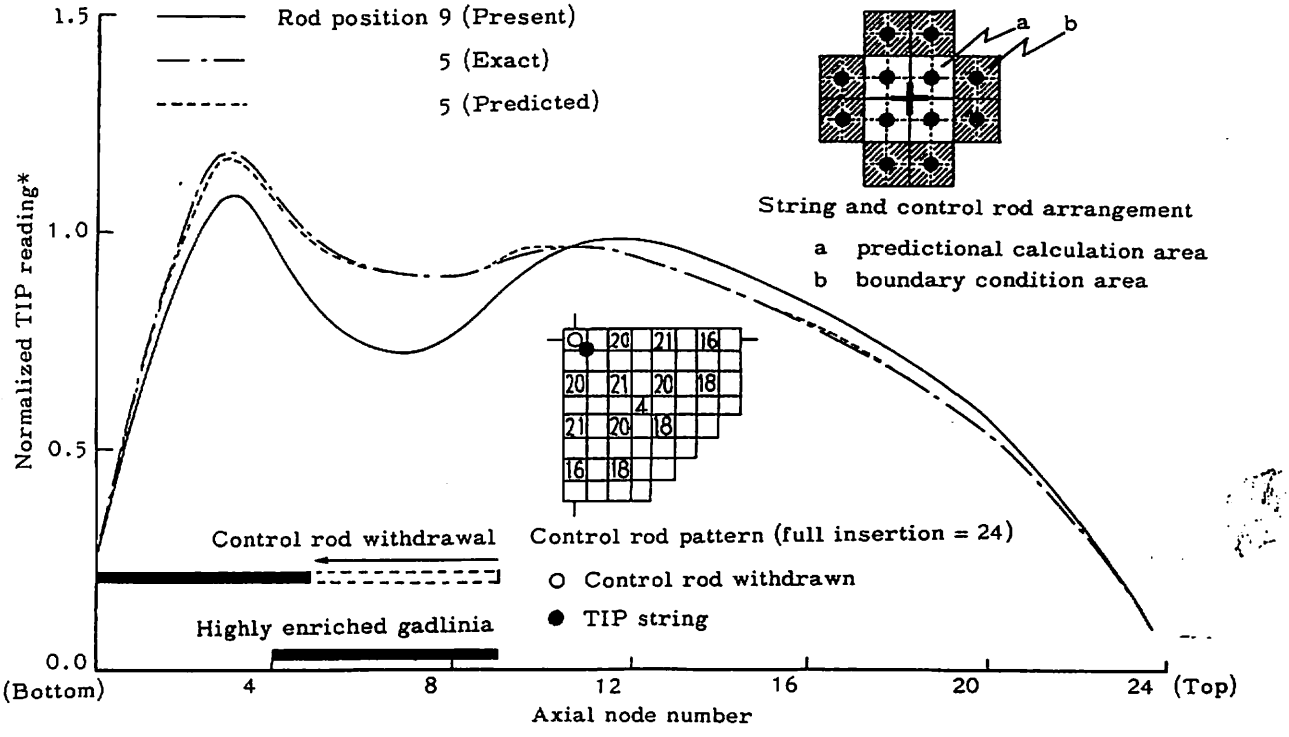
$$S_b(k) \approx S^0_b(k) + \frac{W^H_a}{\dfrac{1}{k^0_{\infty b}(k)} - 1 + 2W^H_b(k)}\,[S_a(k) - S^0_a(k)] \quad (3)$$

Figure 1 shows one of the numerical results. Prediction error is about 2%.

A similar technique has been used in predicting the power distribution change due to flow rate change or xenon transient. In this case, it is assumed that the neutron reflection rate mentioned above remains the same. Prediction error obtained is about 3%.

Redistribution of the monitored channel power to the individual assembly power uses a method similar to that used in the present periodic core performance calculation.

In conclusion, the method presented here predicts power changes encountered in the BWR startup operation with reasonable accuracy within a short computing time and a small core memory and, thus, it is suitable for on-line use.




*Normalized by TIP reading for average power density at rated power state with no void and no adjacent control rods.

Fig. 1. An example of TIP reading prediction.

1. D. L. DELP et al., "FLARE, A Three Dimensional Boiling Water Reactor Simulator," GEAP-4598 (1964).

## 8. Optimal Pin Enrichment Distributions: The Combinatorial Case, *E. Y. Lim (Stanford Univ), A. Leonard (NASA-Ames)**

Determination of the fuel pin enrichment distribution which yields the best approximation to a uniform power distribution in an N-pin fuel bundle may be formulated as the optimization problem

$$\min_{\epsilon \geq 0} y(\epsilon) = \min_{\epsilon \geq 0} || P(\epsilon) - \frac{1}{N} u u^T P(\epsilon) ||^2 \qquad (1a)$$

subject to

$$h(\epsilon) = u^T \epsilon - N \times (\text{average enrichment}) = 0 \ , \qquad (1b)$$

where $u^T = (1, \ldots . 1)$ and $\epsilon$ and $P(\epsilon)$ are N-vectors whose i'th components represent, respectively, the enrichment and resultant power in the i'th pin. In earlier work, N distinct enrichment types were permitted.[1] This continuous problem is designated P1 and its solution is $\epsilon^*$. This paper discusses the more realistic combinational problem which allows only M(<N) enrichment types. The assignment of M enrichment types to N pins is described by the N × M matrix D, where

$$D_{ij} = \begin{cases} 1 & \text{if the j'th enrichment type is} \\ & \text{assigned to the i'th pin, and} \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

and $\epsilon = D\hat{\epsilon}$, where $\hat{\epsilon}_j$ is the j'th enrichment type. The set $\Omega$ of D matrices has $\sim M^N/M!$ distinct elements.

When D is known, we formulate problem P2 as

$$\min_{\hat{\epsilon} \geq 0} \phi(\hat{\epsilon}) = \min_{\hat{\epsilon} \geq 0} y(D\hat{\epsilon}) \qquad (3a)$$

subject to

$$g(\hat{\epsilon}) = u^T D \hat{\epsilon} - N \times (\text{average enrichment}) = 0 \ . \qquad (3b)$$

The optimal solution $\hat{\epsilon}^*$ in P2 is obtained by modifying the projected gradient method of P1. Computational experience shows that the initial guess, $\hat{\epsilon}^0 = [D^T D]^{-1} D\epsilon^*$, is a good approximation to $\hat{\epsilon}^*$ and usually $\phi(\hat{\epsilon}^0) < 1.1\, \phi(\hat{\epsilon}^*)$. Using the response matrix technique[1,2] to evaluate $P(D\hat{\epsilon}^r)$, computation times on the order of one minute are required for an IBM 360/67 when $30 \leq N \leq 40$.

When D is unknown, we must select an optimal matrix D* from $\Omega$ and also the corresponding $\hat{\epsilon}^{**}$ such that the solution $\epsilon^{**} = D^*\hat{\epsilon}^{**}$ to this nonlinear mixed integer programming problem P3 satisfies

$$y(\epsilon^{**}) = y(D^*\hat{\epsilon}^{**}) = \min_{D, \hat{\epsilon}} y(D\hat{\epsilon}) \ . \qquad (4)$$

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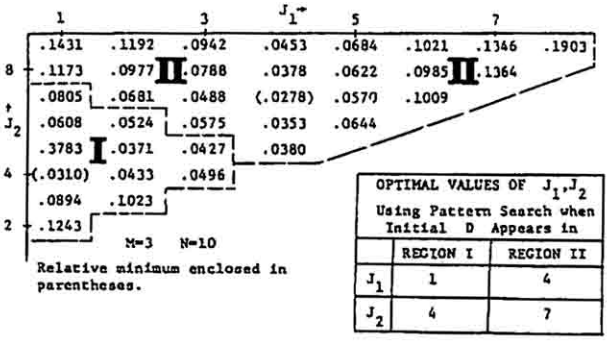


OPTIMAL VALUES OF $J_1, J_2$ Using Pattern Search when Initial D Appears in

| | REGION I | REGION II |
|---|---|---|
| $J_1$ | 1 | 4 |
| $J_2$ | 4 | 7 |

Fig. 1. Enumeration of $y(D\hat{\epsilon}^*)$ assuming $\hat{\epsilon}^{**}$ satisfies ordering defined by $\hat{\epsilon}^*$.

Enumeration or branch and bound techniques using P2 as a suboptimization algorithm will, theoretically, yield $\epsilon^{**}$, but CPU times for all except very small N are staggering. For computational economy, we introduce an assumption which reduces the search for D* to a small subset $\Omega_c \subset \Omega$. Suppose the components of $\epsilon$ in P1 were numbered so that $\epsilon^*$ satisfies $\epsilon_i^* \leq \epsilon_{i+1}^*$ for $i = 1,2 \ldots N-1$. We now assume that $\epsilon^{**}$ also satisfies $\epsilon_i^{**} \leq \epsilon_{i+1}^{**}$ $i = 1,2 \ldots N-1$. This assumption, which has been proven to be valid if the objective function has spheroidal contours, is equivalent to seeking the M - 1 integers $J_1, J_2 \ldots J_{M-1}$ such that

$$\begin{aligned} \epsilon_1^{**} = \epsilon_2^{**} = \ \ldots \ = \epsilon_{J_1}^{**} = \hat{\epsilon}_1^{**} \\ \vdots \qquad\qquad\qquad\qquad \vdots \\ \epsilon_{J_{M-1}+1}^{**} = \epsilon_{J_{M-1}+2}^{**} = \epsilon_N^{**} = \hat{\epsilon}_N^{**} \end{aligned} \qquad (5)$$

and $1 \leq J_1 < J_2 \ldots < J_{M-1} < N$.

Since $\Omega_c$ has $\sim N^{M-1}/(M-1)!$ elements, enumeration-type methods become feasible with $M \leq 4$ and $N \leq 40$. Figure 1 shows $y(D\epsilon^*)$ for D in $\Omega_c$ when M = 3 and N = 10. Work has also been done in adapting a Hooke-Jeeves type of pattern search[3] to find D* in $\Omega_c$. However, there may exist several relative minima in $\Omega_c$ and the D* obtained may depend on the initial D as shown in Fig. 1. Repeated application of this pattern search with randomly chosen initial D's increases the probability of obtaining a global minimum.

1. E. Y. LIM and A. LEONARD, "Optimal Pin Enrichment Distributions by Gradient Search Techniques," *Trans. Am. Nucl. Soc.*, 19, 172 (1974).

2. C. T. McDANIEL, "A Two Dimensional Few Group Response Matrix Calculation Method for Flux and Reactivity," *Proc. Topical Mtg. Computational Methods in Nuclear Engineering* (Apr. 1975).

3. R. HOOKE and T. A. JEEVES, "'Direct Search' Solution of Numerical and Statistical Problems," *J. Assn. Comp. Mach.*, 8, 212 (1959).