BOILING WATER REACTOR CONTROL ROD PROGRAMMING USING HEURISTIC AND MATHEMATICAL METHODS

REACTORS

KEYWORDS: *BWRs, control elements, programming, algorithms, optimization, power distribution*

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OPROD, a computer code for automatic generation of control rod programming that has successfully been applied to an older boiling water reactor (BWR), has experienced some difficulties when applied to a BWR of larger power density and stronger heterogeneity. To improve the performance, a heuristic algorithm that is derived from accumulated experience has been introduced to search for a feasible rod pattern that satisfies all constraints. Application of this algorithm to an initial cycle of an 800-MW(electric) BWR of high heterogeneity has been very successful. It has been demonstrated that the proposed algorithm is capable of finding a feasible rod pattern, even starting from an all-rods-out pattern.

Some improvement was also made in the method of approximation programming (MAP) algorithm. The temporal constraint relaxation method is shown to be effective in finding an optimal control rod pattern in MAP starting from a guess pattern that is not feasible.

INTRODUCTION

Core management is one of the areas of common interest to both utilities and vendors. Considerable cost and manpower are now expended in this area to develop programs and to explore methods for more efficient operation of power reactors. Since vast numbers of problems are categorized to fall in this area, the normal approach is to divide a specific problem into a hierarchy level scheme.

Power distribution control is one of the major areas of concern due to economic and safety requirements of large cores. Current operational constraints on the local power density and its rate of change to secure fuel integrity make this problem challenging. Various techniques have been devised and applied to solve the problems of power distribution control. Previous works and the present state of the art have been reviewed by Karppinen.[1]

This paper discusses a static power distribution control problem of a boiling water reactor (BWR), and is an extension of our previous work.[2] Practicality is emphasized rather than theoretical rigorousness, and for this purpose, a three-dimensional BWR simulator is used as a basic tool for core management engineers. OPROD, a computer code for automatic generation of control rod programming for BWRs, is one code in our core management system,[3] which has been successfully applied to a commercial 460-MW(electric) (40.6 kW/ℓ) BWR for more than five operating cycles. It has dispensed with manual search and has facilitated prompt responses to unexpected changes in operating conditions.

Since there are many variables and constraints that are closely interrelated to each other, the size of the problem must be reduced to a manageable level. Figure 1 shows the objective of the present paper and the three phases employed in OPROD. The first is to set up basic operating policy. This includes the concept of control rod grouping, pattern change, etc. These are input to OPROD and are not optimized by the code itself. The second phase is to prepare an initial guess of the control rod pattern, which is also an important input to OPROD. The third phase is what OPROD actually does. It tries to optimize the rod pattern by a mathematical programming method.

During recent application to a larger BWR loaded with nonuniformly distributed gadolinia fuel, some difficulties have been encountered that are mostly due to stronger heterogeneity and increased power density. The problem seemed to lie in finding a feasible solution, not an optimal one. Even a carefully chosen initial guess pattern did not necessarily fall in

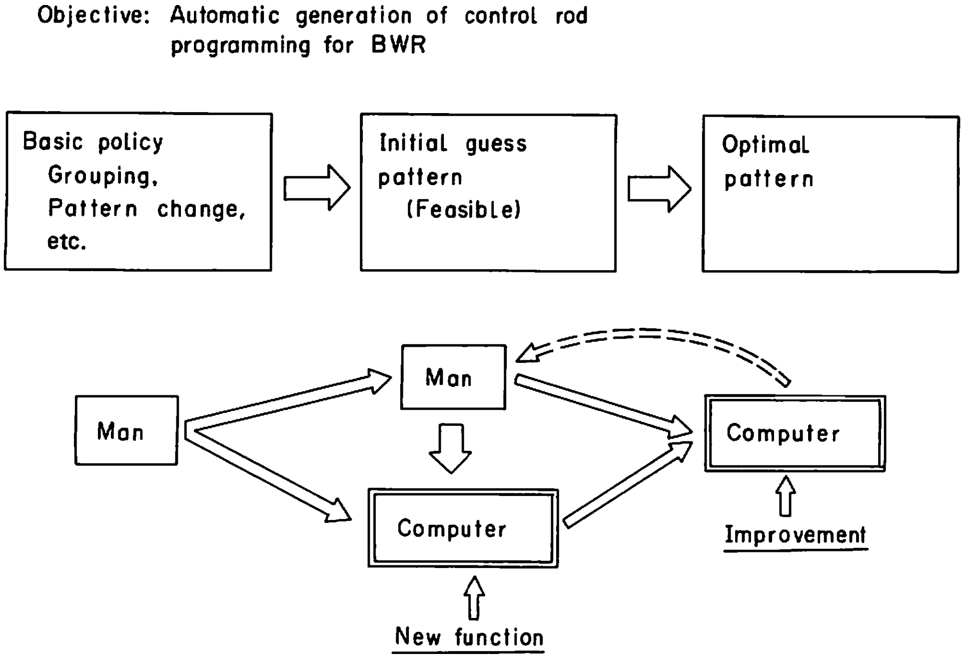

Fig. 1. Objective of present paper and three phases employed in OPROD.

a feasible region, and repetition of phases two and three was often required. Stronger heterogeneity together with the reduced thermal margin brought about by the increased power density made convergence worse.

To improve the performance, a new function has been added[4] to automate the generation of a feasible guess pattern for which all of the constraints are satisfied, and this function has been coupled to the original optimization routine, in which a small improvement has also been implemented. Hence, the main objective of the present work is to computerize phase two.

PROBLEM ENCOUNTERED IN MAP APPLICATION

The method used in the original OPROD is summarized by the simplified flow diagram given in Fig. 2. The meanings of the parameters are given in the Nomenclature on p. 100. There are two hierarchical optimization loops. In the inner loop, the control rod pattern is optimized to minimize the difference from a target power distribution. This process is repeated until the end-of-cycle (EOC) by increasing fuel burnup and exchanging the rod pattern. In the outer loop, the target power distribution, fixed in the inner loops, is revised to minimize the rods remaining in the core at EOC.

One of the characteristics of the approach is that, as stated earlier, a three-dimensional BWR simulator[5] is used as a system equation with all of the constraints explicitly taken into account. The method of approximation programming[6] (MAP) is used to solve the inner loop problem. Sensitivities to the constraints and the performance index of a small amount of movement of individual rods are calculated periodically, and linear programming is repeatedly used.

The difficulty is that for a BWR of higher power density, which means smaller thermal margin and stronger heterogeneity, a linearity assumption does not hold for a rod movement of reasonable range. Accordingly, convergency becomes worse, and there arises a possibility of divergency in some cases. The success of the search strongly depends on the initial guess prepared.

Figure 3 is a typical example showing how bad the convergency is when MAP is applied to a BWR of high heterogeneity. The vertical axis in the upper graph is the performance index, which is a measure of the difference from the target power distribution (given in Fig. 2), while the lower graph shows the maximum linear heat generation rate (MLHGR). The horizontal axis for both is the number of linear programming calculations.

As can be seen from these plots, the initial guess pattern does not satisfy the constraint on MLHGR [the limit was set at 55.8 kW/m (17.0 kW/ft)], although the performance index is not that bad. After calculating the gradients of g (constraints) and J (performance index) 8 times, and solving the linear programming 13 times, the solution finally converges to an optimal pattern. This demonstrates the fact that the problem seems to lie in finding a feasible solution. In this case, the rod densities of the second and third

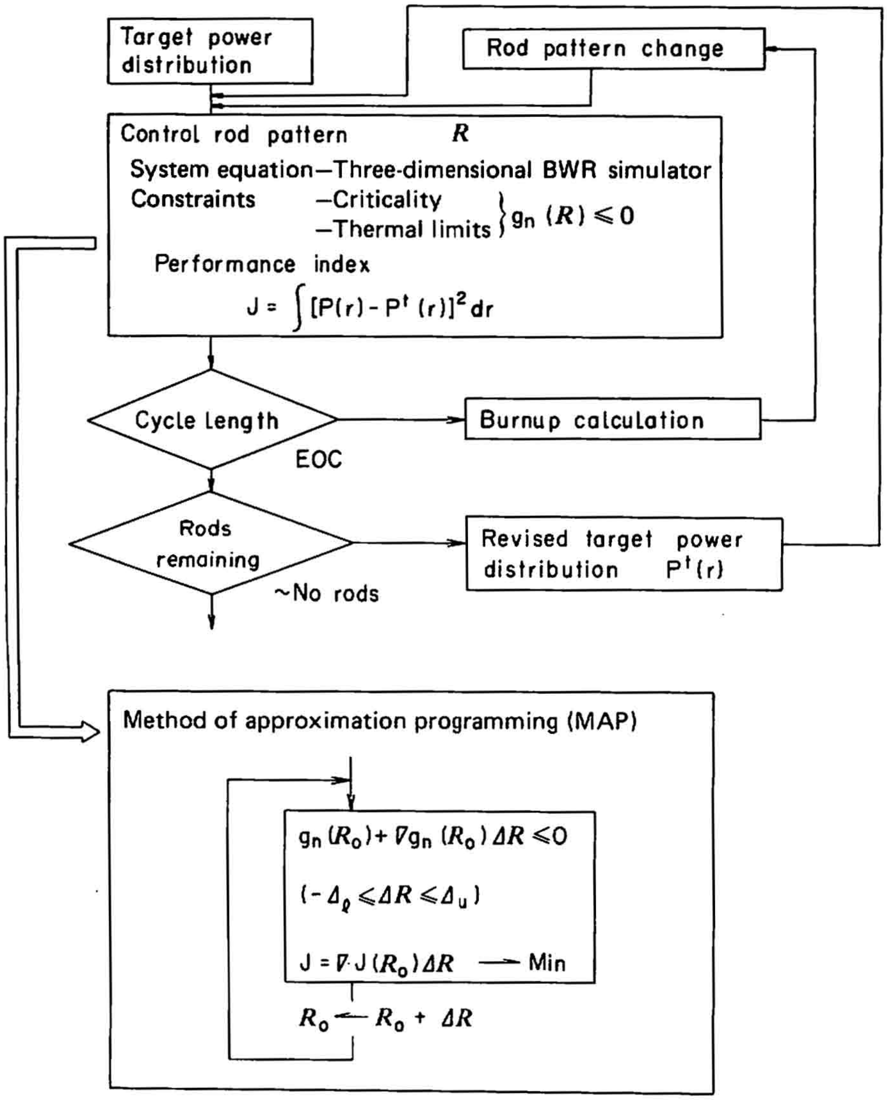

Fig. 2. Simplified flow diagram of OPROD.

layers (rings) are reversed, and it has taken time to switch this density distribution by the MAP algorithm. It has been observed that for most of the cases that were successful when starting from an unfeasible guess pattern, the feasible pattern was found by chance through the accumulation of repeated linear programming results. Hence, it is very desirable to develop a method for finding a feasible solution within a reasonable number of three-dimensional power distribution calculations.

METHOD DEVELOPMENT AND IMPROVEMENT

Implementation of Heuristic Algorithm

A new function is added to automatically search for an initial guess pattern. The objective is to find a feasible solution and to search for a rod pattern that improves the core characteristics with respect to the severest constraint, even if starting from an all-rods-out pattern. It is felt necessary that the approach be capable of grasping the core as a whole and, in that way, to determine a pattern that is more rational and geometrically better arranged than those obtained by other mathematically programmed methods. This routine is placed before the MAP routine and thus is called pre-MAP for convenience. The method is categorized as heuristics and is based on two principles:

1. Control rods are grouped deep and shallow. However, effort is made to search for a rod pattern using deep rods alone, thereby aiming at burning the lower part of the core as much as possible.

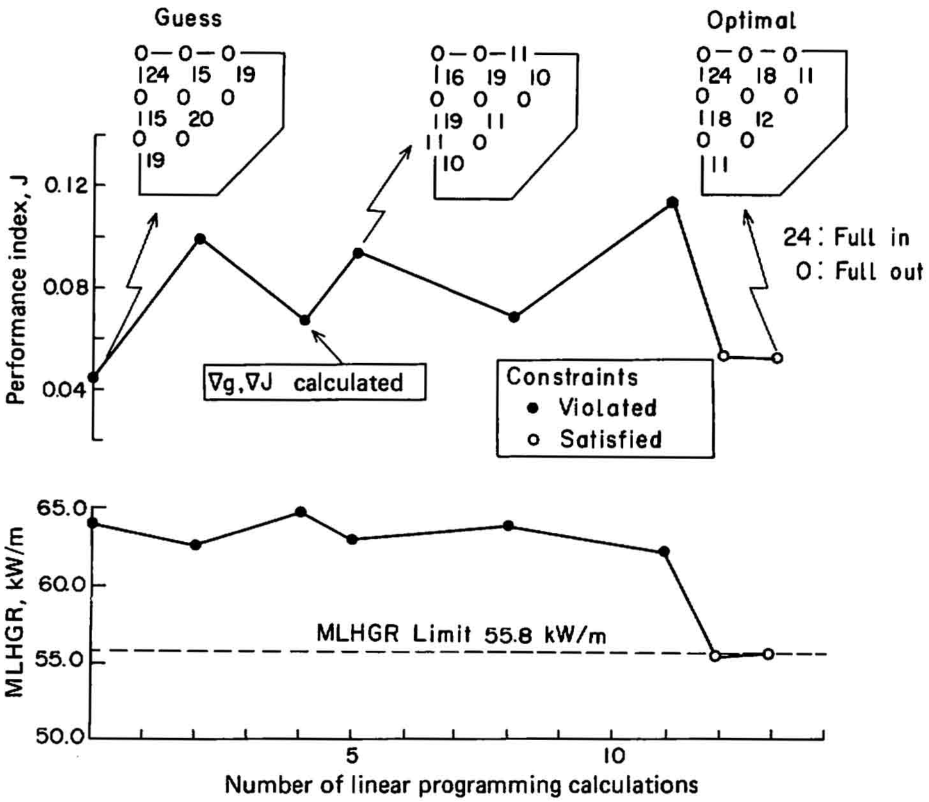

Fig. 3. An example of convergence of MAP applied to a BWR of high heterogeneity.

2. The algorithm is basically a repetition of searches with two degrees of freedom. The control rods are always split into two subgroups, and the rods within each subgroup are moved in the same direction for the same distance. The two subgroups are redefined in going from the rough adjustment of step 1 to the fine adjustment of step 3 in accordance with the rod pattern improvement.

Step 1 is criticality and deep rods position search. All of the control rods to be used in the search are temporarily classified as deep and shallow, and placed in one of two subgroups. The deep rods are used to decrease reactivity and the shallow to increase it, as shown in Fig. 4.

Normally, starting from the all-rods-out pattern, criticality can be achieved by deep rods alone. However, the power distribution at this stage may not be sufficiently good. Location of gross power peaking is searched, and if the rod nearest to this location is a deep rod, the search goes to step 2. If it is a shallow rod, this shallow rod is reclassified as deep and each of the two subgroups is redefined. Criticality is again searched using the deep rods alone. This procedure is repeated until the nearest rod to the location of gross power peaking is a deep rod.

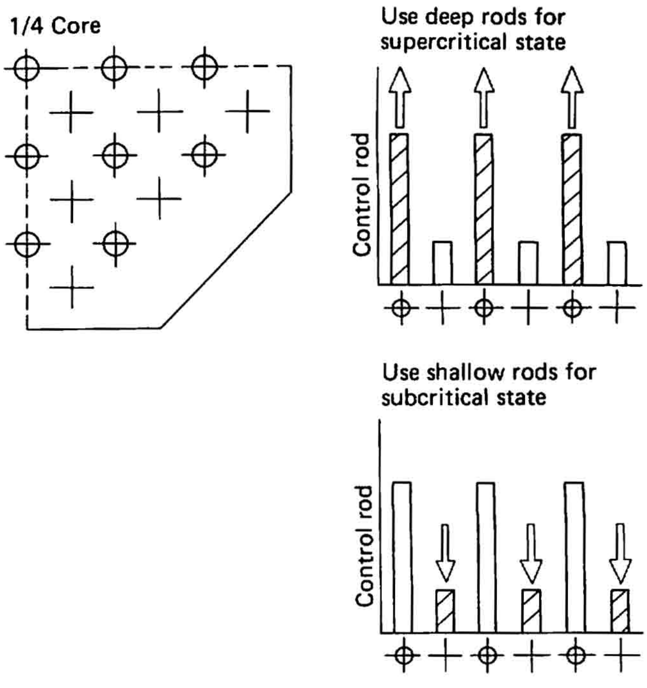

Fig. 4. Pre-MAP algorithm: step 1—criticality and deep rods position search. Subgroup 1: ⊕ = deep rods; subgroup 2: + = shallow rods.

Step 2 attempts to improve core characteristics by deep rods search, as follows (Fig. 5). First, in step 2-1, the deep rods are divided into as many groups as necessary, out of which two subgroups are formed. A search is made for a rod pattern that gives the best core characteristics under the constraint of criticality. Here, the core characteristics are represented as one of the following quantities: gross power peak, channel power peak, MLHGR, minimum critical power ratio (MCPR), and minimum critical heat flux ratio (MCHFR). Normally, MLHGR is chosen as representative of the core characteristics.

Then, in step 2-2, ranking is made of the core characteristics throughout the regions associated with these groups, and two new subgroups are formed by selecting the highest and the lowest ranking groups. The search is again made for the best rod pattern. If the rod pattern obtained is an improvement over the starting pattern, this step is repeated. If not, the search goes to step 2-3. A new subgroup is formed by combining the highest and the second highest ranking members and the search is continued for the best pattern. If the best pattern obtained is an improvement, the search goes back to step 2-2, and the above procedure is repeated. If not, the search goes to step 3. Throughout step 2, no shallow rods are used in the search. The converged pattern results in a radially flattened power distribution.

Step 3 is fine adjustment of deep and shallow rods. The shallow rods are introduced here for the

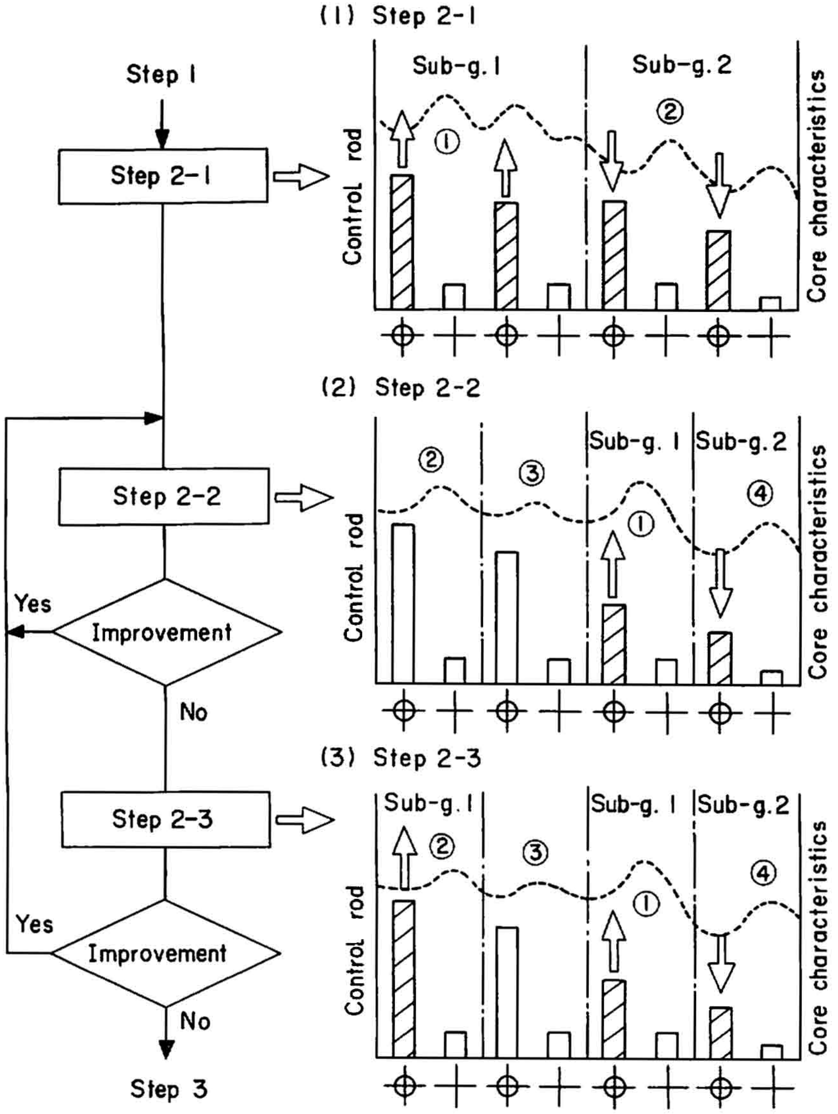

Fig. 5. Pre-MAP algorithm: step 2–improvement of core characteristics by deep rods search (radial direction control). The circled numbers represent core characteristics ranking. Here, ⊕ = deep rod; + = shallow rod.

first time to be searched. The deep and the shallow rods nearest to the location of the worst core characteristics are formed into subgroups 1 and 2. These rods are adjusted to further improve the core characteristics. This procedure is given in Fig. 6.

If the location of the worst core characteristics has changed by the rod adjustment, two new subgroups are redefined according to the new location. The procedure is repeated until no further improvement is expected.

As is evident from the above-mentioned algorithm, this method always gives a pattern that has improved core characteristics. No oscillation and divergence that have been observed in the MAP algorithm take place. The obtained pattern is rational and easily acceptable from an engineering viewpoint.

Improvement of MAP

The present MAP algorithm includes a routine that automatically adjusts the amount of allowable control rod movement (Δl, Δu) for each linear programming problem. For an initial guess pattern that is not feasible, Δl and/or Δu sometimes become too large for a linearity assumption to be valid. This drawback may be overcome by including a temporal constraint relaxation algorithm such that Δl and/or Δu stay within the region with a valid linearity assumption. Figure 7 shows the conceptual trajectory of optimization process by this algorithm. The relaxed constraints are gradually tightened according to the pattern improvement.

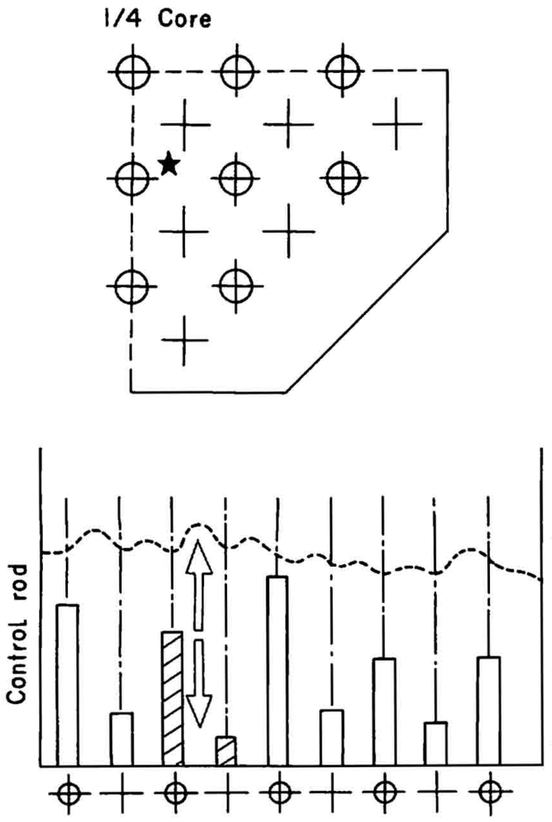

Fig. 6. Pre-MAP algorithm: step 3—fine adjustment of deep and shallow rods. Subgroup 1: deep rod nearest to the location of the worst core characteristics; subgroup 2: shallow rod nearest to the location of the worst core characteristics. Here, ⌖ = deep rod; + = shallow rod; ☆ = location of the worst core characteristics.

RESULTS AND DISCUSSION

Heuristic Method—Pre-MAP

To examine the validity of the pre-MAP routine, the initial cycle of an 800-MW(electric) BWR (47.1 kW/ℓ) having three different 8 × 8 enriched fuel assemblies with axially distributed gadolinia has been chosen. The core map and the fuel assemblies are shown in Fig. 8. Natural uranium fuel assemblies are placed in the core periphery, and high and intermediate enriched assemblies are scattered in the core center region. The core configuration is thought to be highly heterogeneous together with axially distributed gadolinia.

Figure 9 shows an example of the convergency process of pre-MAP. The objective in this example is to minimize MLHGR, with the limit set at 40.4 kW/m (12.3 kW/ft). In step 1, starting from the all-rods-out pattern, criticality has been achieved by the deep rods alone. Next, in step 2, flattening of radial power distribution is promoted and accordingly MLHGR is improved. In step 3, MLHGR is further improved. The obtained rod pattern does not include shallow rods.

The variation of two other characteristics, MCPR and MCHFR, is also plotted in the same figure. These two are also improved in general. It has been possible

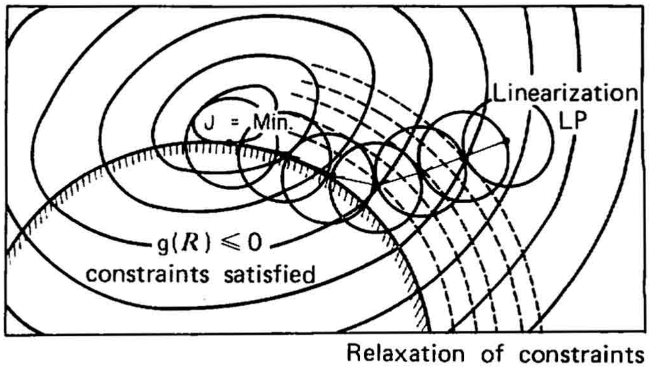

Fig. 7. Conceptual trajectory of optimization process by the improved MAP (constraint relaxation method).

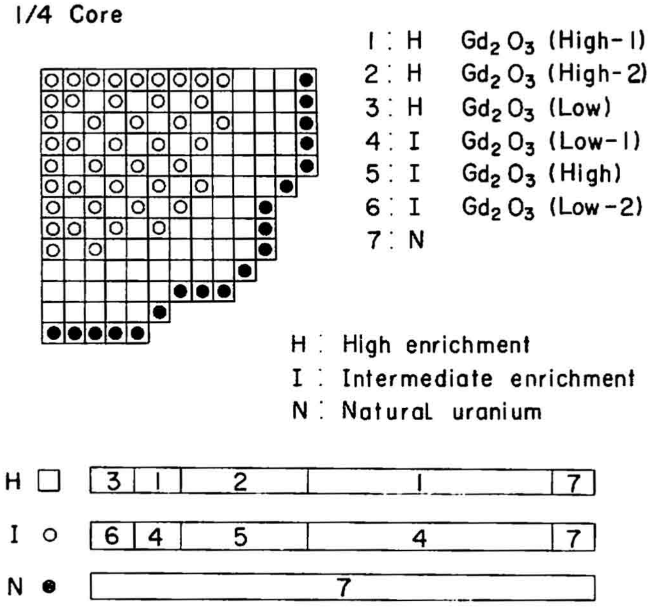

Fig. 8. Reference core specifications of a BWR of high heterogeneity. Total reactor power, 800 MW(electric); total coolant flow, 35.13 Gg/h.

to get all of these three well inside the limits. It should be mentioned, however, that these constraints tend to compete with each other near the end of the search. For example, MCHFR has become worse in going to step 3, whereas MLHGR has been improved.

Figure 10 shows the core characteristics throughout the whole cycle obtained by pre-MAP. All of the calculations were started from all-rods-out guess patterns. Only the initial values of step 2 and the final results of step 3 are shown here. As in Fig. 9, the objective was again to minimize MLHGR. Some of the initial values of step 2 do not satisfy all of the constraints, but it has been possible to search for a series of rod patterns that do satisfy all of the constraints throughout the cycle using the proposed heuristic algorithm.

The final step of optimization is shown in Fig. 11. Results of two exposure steps are shown here. The objective is now to minimize the difference from a target power distribution, which is taken to be a Haling power distribution in this example.

Since the starting guess is the converged solution of pre-MAP that is feasible and satisfies all of the constraints, MAP is straightforward, and there is no

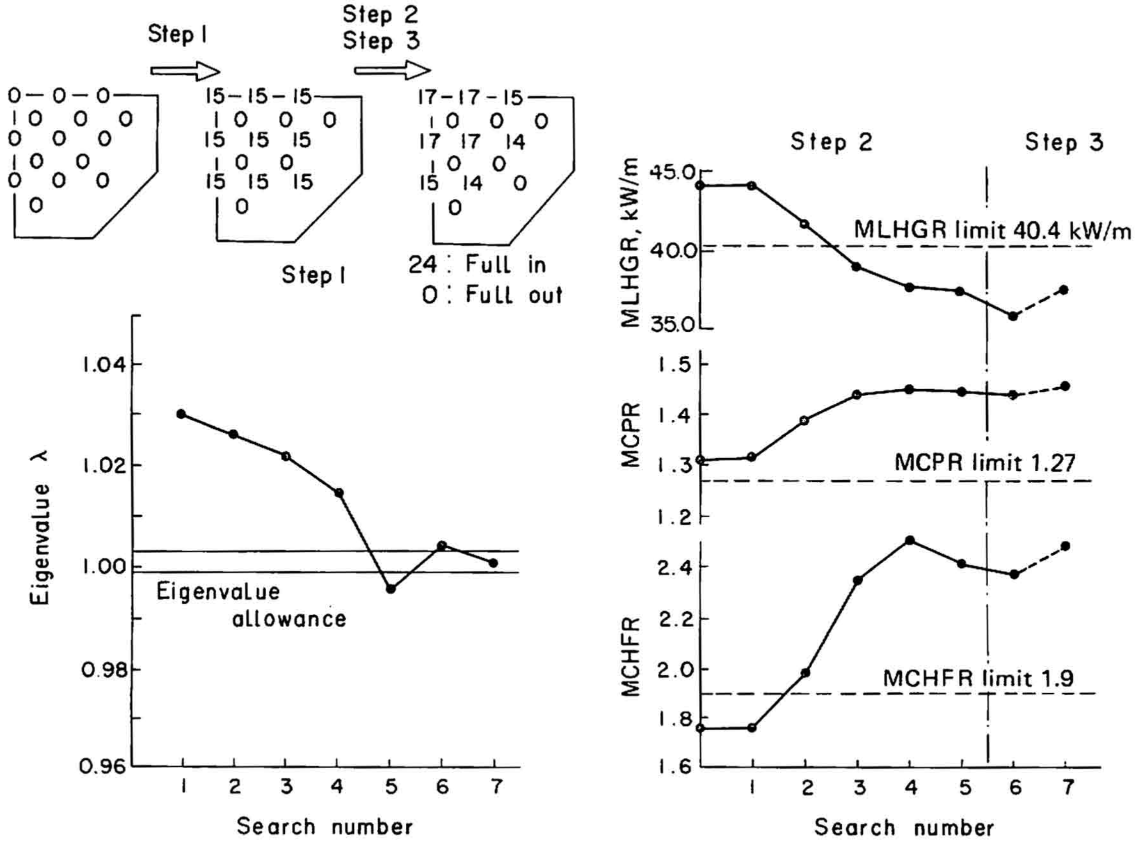

Fig. 9. An example of the convergence process of pre-MAP.

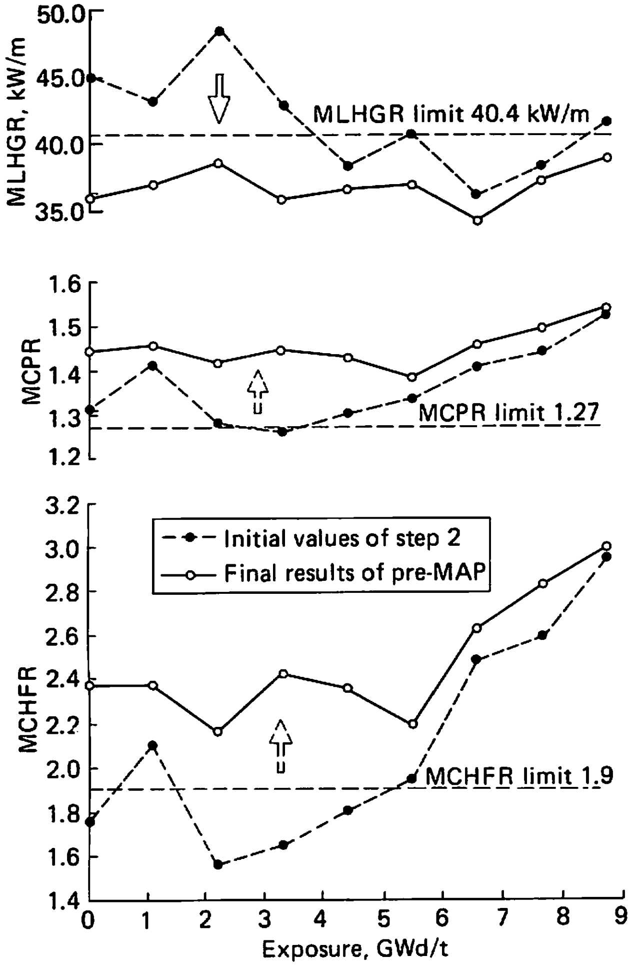

Fig. 10. Improvement of core characteristics for a whole cycle by pre-MAP.

problem in finding a converged solution. In the lower example, sensitivities were calculated at the starting pattern, and linear programming was solved. The performance index was reduced, and the constraints were satisfied. Since the first trial was a success, linear programming was solved again with increased rod movement allowance using the same sensitivities. The result again was successful, and the rod movement allowance was further relaxed. This time, the solution did not satisfy some of the constraints, although the performance index has been reduced. The base point was moved from the starting pattern to the second trial (search number 2), at which the sensitivity coefficients were recalculated. The next two trials were failures, and the rod pattern obtained at the second trial was concluded to be optimal. The optimal pattern is not much different from the pre-MAP solution. The results at other exposure points are almost the same. In this way, it has been demonstrated that the proposed method is successful in eliminating the laborious preparation of an initial guess pattern. The elimination of the wasteful trial search results from using the heuristic algorithm that has been derived from accumulated experience.

Temporal Constraint Relaxation Method—Improved MAP

It was not necessary to use this routine for the reference core selected in the previous section because pre-MAP was successful in finding a feasible solution. To see how this temporal constraint relaxation method works, a simple 4 X 4 core was chosen from the central region of the reference core with a repetitive boundary condition. This core model simulates the central region of a larger core. The solid line in Fig. 12 is a case where the old MAP has not converged. The initial guess pattern does not satisfy the constraints. Linear programming was done with the rod movement allowance $\delta R = 4$. (The core was divided into 24 segments in an axial direction.) There was no feasible solution of linear programming, so δR was increased to 16, which gave the first feasible solution in the linearized model. Recalculations of the core characteristics with the obtained rod pattern showed violation of the constraints and an increase of performance index. The rod movement allowance was reset at 4, and the calculation continued. The solution diverged after three successful linear programming calculations. The dotted line shows a case where an optimal rod pattern could be found by the improved MAP. Starting from the same initial guess pattern, linear programming had a feasible solution with $\delta R = 4$, and the constraints relaxed by $n = 1$ on the first time. Recalculation of the core characteristics showed an increase of the performance index and satisfaction of the constraints. The constraints were tightened back to the original values ($n = 0$). Using new sensitivity coefficients, linear programming had a feasible solution, but the recalculated core characteristics showed violation of the constraints. Since the previous step was successful, the base point was moved and new sensitivity coefficients were calculated. The next linear programming had no feasible solution, and δR was reset at 1. The following three trials with constraints relaxed (n = 1, 2, and 3) were unsuccessful. Linear programming obtained a feasible solution with $n = 4$ but the constraints were found to be violated by the recalculation (number of successful linear programming solutions = 3). Repeating the same procedure, the solution finally arrived at a feasible solution that satisfies the constraints. The constraints were tightened back again to the original values ($n = 0$), and the sensitivity coefficients were recalculated. Linear programming was done for the same δR. The solution was feasible, and since the performance index had not been improved, it was concluded that an optimal has been obtained.

The above example shows that the constraint relaxation method works well starting from an unfeasible pattern, but it is felt desirable that the starting pattern is feasible.

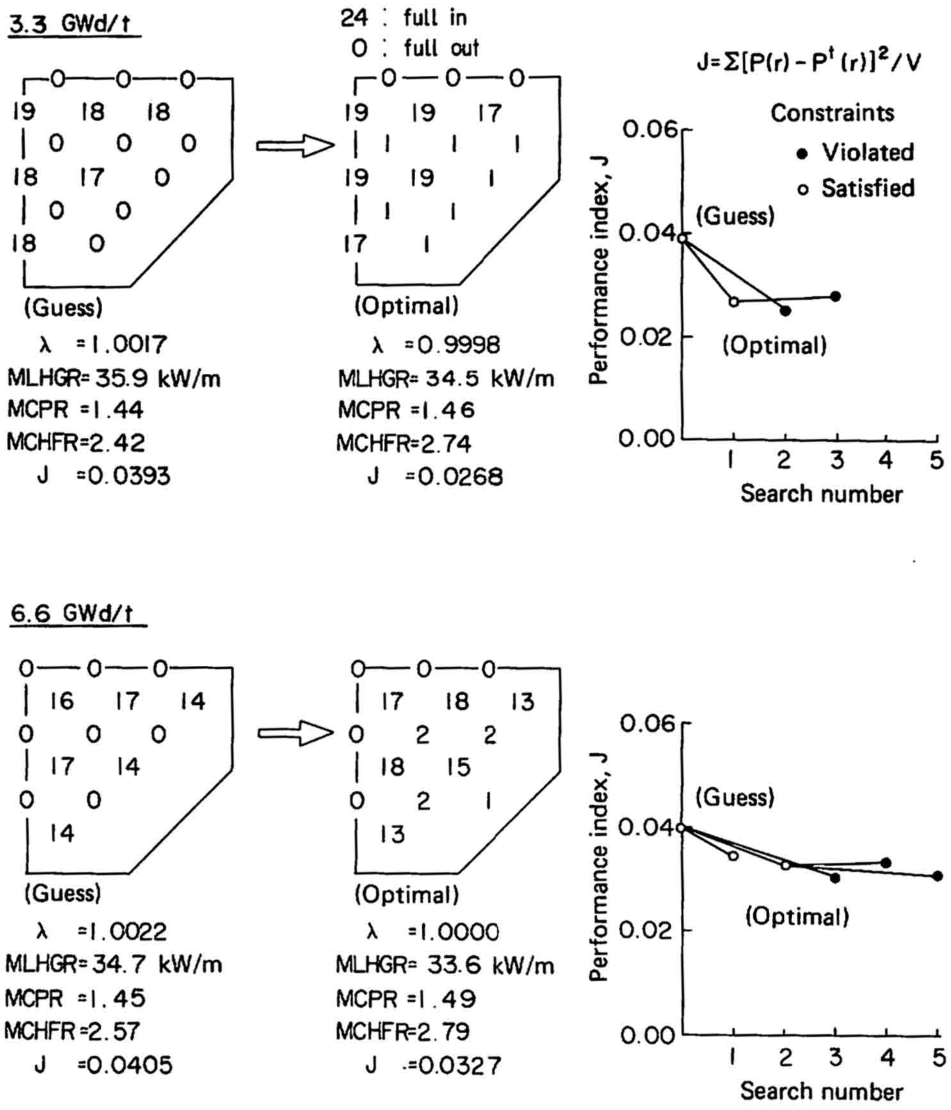

Fig. 11. Examples of combination of pre-MAP and MAP.

CONCLUSION

The degree of difficulty of core management calculations depends on core design. OPROD, a computer code for automatic generation of control rod programming for BWRs, has successfully been applied to an older commercial BWR, but has experienced some difficulties during applications to a newer and larger BWR. These were mostly due to stronger heterogeneity and increased power density.

To improve the performance, a new heuristic algorithm was developed to automatically search for an initial guess pattern that satisfies all of the operating constraints, and this function was added to OPROD. This algorithm is based on repetition of the search with two degrees of freedom and with priority given to deep rods. The MAP algorithm originally used in OPROD was also improved by employing the temporal constraint relaxation method.

Heuristic algorithm was applied to the initial cycle of a 800-MW(electric) BWR having three different enriched fuel assemblies with axially distributed gadolinia. It has been possible to find a series of feasible control rod patterns starting from an all-rods-out pattern throughout the cycle. The algorithm has succeeded in dispensing with laborious preparation of an initial guess pattern.

Improvement of the MAP algorithm also showed merits. The temporal constraint relaxation method has been demonstrated to work well starting from a guess rod pattern that is not feasible.

With the above improvements, OPROD should become a very useful tool in core management of BWRs.

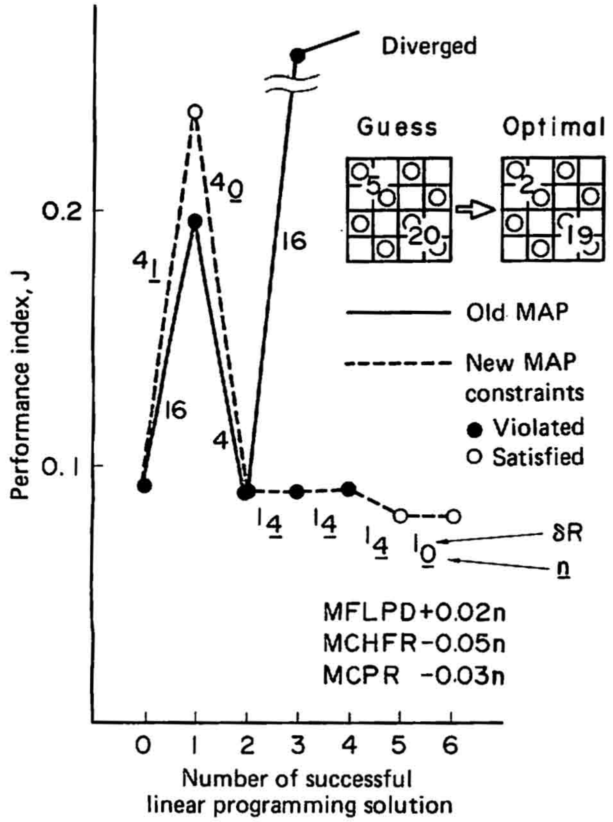

Fig. 12. Convergence test of the improved MAP.

NOMENCLATURE

$\boldsymbol{R}$ = control rod pattern ($\boldsymbol{R}_0$ = initial value)

$\Delta \boldsymbol{R}$ = control rod pattern movement

Δu = upper limit of $\Delta \boldsymbol{R}$ for rod insertion

Δl = lower limit of $\Delta \boldsymbol{R}$ for rod withdrawal

$P(r)$ = calculated power distribution

$P^t(r)$ = target power distribution

$g_n(\boldsymbol{R})$ = criticality and thermal constraints

$$\begin{cases} \text{constraints satisfied} & g_n(\boldsymbol{R}) \leqq 0 \\ \text{constraints violated} & g_n(\boldsymbol{R}) \geqq 0 \end{cases}$$

$\nabla g_n(\boldsymbol{R})$ = gradient of $g_n(\boldsymbol{R})$

J = performance index $\int [P^t(r) - P(r)]^2 dr$

$\nabla J(\boldsymbol{R})$ = gradient of $J(\boldsymbol{R})$

λ = eigenvalue

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