A METHOD FOR GENERATING A CONTROL ROD PROGRAM FOR BOILING WATER REACTORS

REACTORS

KEYWORDS: *BWR-type reactors, control elements, optimization, burnup, power distribution, O codes, fuel management*

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The OPROD computer code has been developed to generate a long-term control rod program, a series of control rod patterns that optimizes a cycle length within various operational constraints. In the algorithm, the optimization problem is decomposed into two hierarchies. In the inner loop, a time-invariant target power distribution is assumed, and a control rod pattern is determined so as to best fit the power distribution to the target within the constraints at each burnup step. The target is then improved in the outer loop to achieve a longer cycle length. The code consists of two major parts: a three-dimensional boiling water reactor (BWR) core simulator and MAP, the method of approximate programming. It readily generates a long-term control rod program of BWRs without trial search by core-management engineers. The OPROD has therefore facilitated prompt response to varying operating conditions and the investigation of a conflicting relationship between the thermal limitation and the cycle length.

INTRODUCTION

Long-term operational strategy for power reactor cores is an important field of research because it contributes directly to the safety and economy of nuclear power.

A boiling water reactor (BWR) is equipped with 100 to 200 control rods that are gradually withdrawn from the core during operation to compensate for reactivity depletion. Control rod pattern, i.e., the depth distribution of these rods, determines the power distribution or the burnup rate distribution in the core. The control rod pattern must be scheduled to optimize the cycle length within the requirements to prevent overheating of fuel rods.

Control rod programs so far have been generated by trial methods. Programs are repeatedly set up and tested on a core burnup simulator until a core-management engineer finds the result satisfactory. Such an empirical approach requires much manpower and computation time, because three-dimensional (3D) simulation must be repeated many times for the multistage optimization of this type. Thus, the result sometimes remains far from an optimum. Thermal margin and operational freedom decrease as burnup proceeds, and in extreme cases, a reactor comes to an end-of-cycle (EOC) with some rods partially inserted because further withdrawal violates the thermal limitation. In spite of such drawbacks, core-management engineers have generated control rod programs for actual practice on the basis of their accumulated experiences.

Another group of theorists has been trying to apply optimization techniques to this problem. Terney and Fenech[1] employed dynamic programming and solved the insertion depth of control rods in two regions of a cylindrical core. Wade and Terney[2] applied the maximum principle of a distributed parameter system to a one-dimensional core, and solved the rod density in two regions at two time-steps. Motoda and Kawai[3] developed a geometrical theory by which general relationships among control rods, power distribution, exposure, and cycle length were visualized for a two-region core. Kitamura and Motoda[4] extended this theory to a modal expansion model and reached essentially the same conclusion as in Ref. 3. Suzuki and Kiyose[5] discussed general conditions that an optimal EOC state of a core should satisfy and applied the theory to a two-region problem using the maximum principle. Recently Snyder and Lewis[6] used dynamic programming to solve an axial control problem of the BWR for various cost functions. Although the model was realistic in one dimension, only two control densities were assumed, and their depths were determined as functions of time. A coupled control-refueling optimization problem was discussed by Motoda[7] qualitatively in the burnup space. He further attacked the coupled problem for a one-dimensional multiregion core, finding an optimal loading and an optimal control rod density as functions of space and time.[8] These works have helped to reveal the general nature of the optimum strategy on core control.

Common features of these papers are the simplification of a core model, number of core regions, degrees of control freedom, and quantization of rod densities. It is difficult to generate a rod pattern for an actual 3D core on the basis of these analyses.

It is essential that a 3D model is used as a tool for core-management engineers. The complexity of the model prohibits pursuit of rigorous optimality within reasonable computation time, and one has to be satisfied with a suboptimal solution. In addition, the increase in computation time must be kept to a minimum by various means. Much information from past theoretical works and empirical know-how should be utilized. By incorporating the practical information available, we have developed OPROD, an efficient computing code for generating long-term control rod programs that can be applied to the actual core-management.

LONG-TERM CONTROL ROD PROGRAM

A typical BWR is selected as a reference reactor. Its specification is summarized in Table I. It is composed of 400 fuel bundles and 97 control rods. Their configuration is shown in Fig. 1.

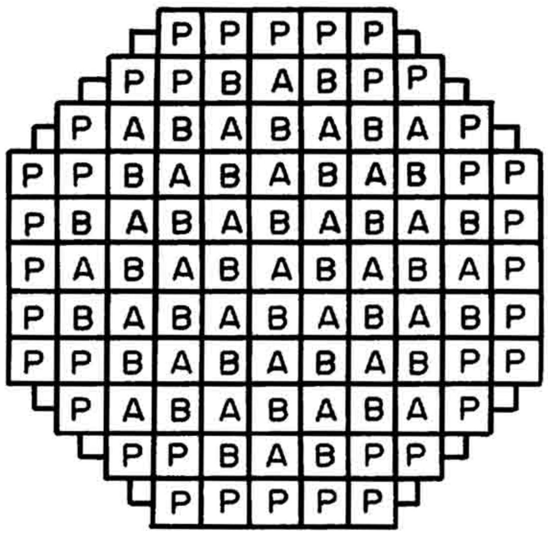
Actually, most control rods are withdrawn at full-power operation; the control fraction is 15% at maximum. In practice, one-third of the rods located at periphery (designated by P in Fig. 1) are always completely withdrawn to flatten the radial power distribution. The remaining central rods are divided into two groups (designated by A and B in Fig. 1). One of the groups is completely withdrawn and the other is partially withdrawn. When rods in group A are inserted, the reactor is said to be operated in "A-pattern." The "B-pattern" has $\frac{1}{4}$ core mirror symmetry, while the A-pattern has 90-deg rotational symmetry in addition. To flatten the exposure distribution, the rod pattern is changed from A to B and B to A at

TABLE I

Data of a Reference Core

Thermal output	1380 MW
Core flow rate	2.2×10^7 kg/h
Core inlet enthalpy	290 kcal/kg
Fuel bundle	
number of bundle	400
core lattice pitch	305 mm
active fuel length	3660 mm
fuel rod array	7×7
cladding o.d.	14.5 mm
fuel enrichment	2.1%
uranium in core	78.2 ton
Number of control rods	97
Number of poison curtains	172

every exposure interval of a certain amount. During the interval, the rods are gradually withdrawn for reactivity compensation. The problem of power distribution control in this period is treated in the "intermediate control rod programming problem," as separate from the "long-term problem." Besides the above-mentioned shim problem during the exposure interval, there are other kinds of rod programmings for startup and pattern change. All these short-term problems can be solved separately from the long-term problem by considering instantaneous operational



A = A group rods
B = B group rods
P = Periphery rods

Fig. 1. Classification of control rods.

constraints. Figure 2 illustrates various kinds of control rod programs.

Contrary to the short-term programs, economy throughout the cycle must be considered for the long-term programming. This is one of the difficulties for the solution of the long-term control rod programming problem.

DEFINITION OF THE PROBLEM

We formulate the optimization problem in the following framework: to find values of control variables that minimize, within constraints, the cost, which is a function of both state and control variables with interrelations that are described by the system equations.

Measure of Performance

The cost function for the core control method should be given by the power and fuel management requirements. At present, the requirements are rather simple because nuclear reactors are generally used as a base load. They are scheduled to be shut down at off-peak seasons. When the refueling scheme is disturbed by unexpected outages, one of the following decisions is made: derating, premature refueling at off-peak season, or continuing full-power operation.[9]

Whatever happens in the future, however, the figure-of-merit for the core control program is the length of reactivity life. The margin in the cycle length will minimize the penalty of derating; it will also minimize the number of new fuel assemblies if a premature refueling takes place.

Constraints

A reactor must be critical at full power

$$\lambda = \lambda_{target} \quad . \tag{1}$$

Linear power density (LPD) is a direct measure of the thermal performance of a fuel rod. The ratio of LPD to the design value, the fraction of limiting power density (FLPD), must be less than a prescribed value everywhere:

$$\mathrm{FLPD}(r) \leq \mathrm{FLPDMAX} \quad . \tag{2}$$

The ratio of critical heat flux to actual heat flux, CHFR, must be larger than a prescribed value everywhere:

$$\mathrm{CHFR}\,(r) \geq \mathrm{CHFRMIN} \quad . \tag{3}$$

Another constraint is for stuck control rods, if any:

$$\text{stuck rod depth: fixed} \quad . \tag{4}$$

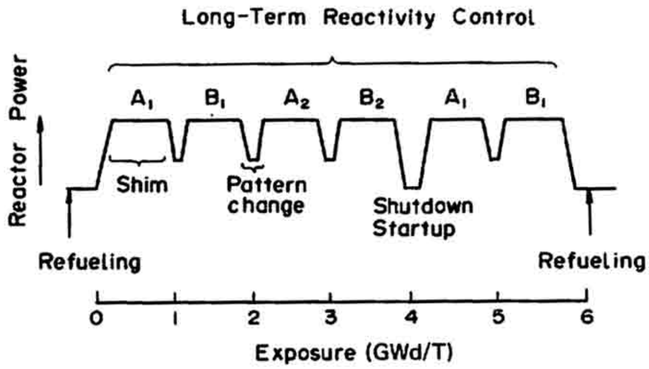


Fig. 2. Classification of control rod programs into four categories.

Control Variables

Core flow, subcooling, system pressure, and control rod pattern determine the core state. However, only the control rod pattern is chosen as the control variable here because the other three variables are always fixed at their rated values for steady-state operation. Flow control is anticipated in the load-following mode operation in the future. It does not appreciably change the power distribution, and the present method is applicable for the flow control operation as well.

System Equation

Any 3D BWR core simulator plays the role of system equations for this problem. It gives the FLPD and CHFR of each fuel rod corresponding to a given control rod pattern. The structure of the simulator is given in Fig. 3. The simulator gives the "segment power" P_{ijk}, where i,j, and k designate node numbers for x, y, and z axes. In practice, a set of ij defines a fuel bundle. The power for each fuel rod is obtained by multiplying a local peaking factor to the segment power. Power-void iteration adds complexity to 3D BWR simulators. Regression formulas, which represent the results of simulations in concise forms, are highly desirable. Nishihara,[10] Kawai and Kiguchi,[11] and Hoshino[12] made regression formulas for a few-region core or a fuel-bundle region to apply space-kinetics and refueling optimization problems. Success in 3D problems is not yet known.

METHOD OF SOLUTION

Problem Size

The number of control rods inserted at full-power operation is 29 (A-pattern) or 32 (B-

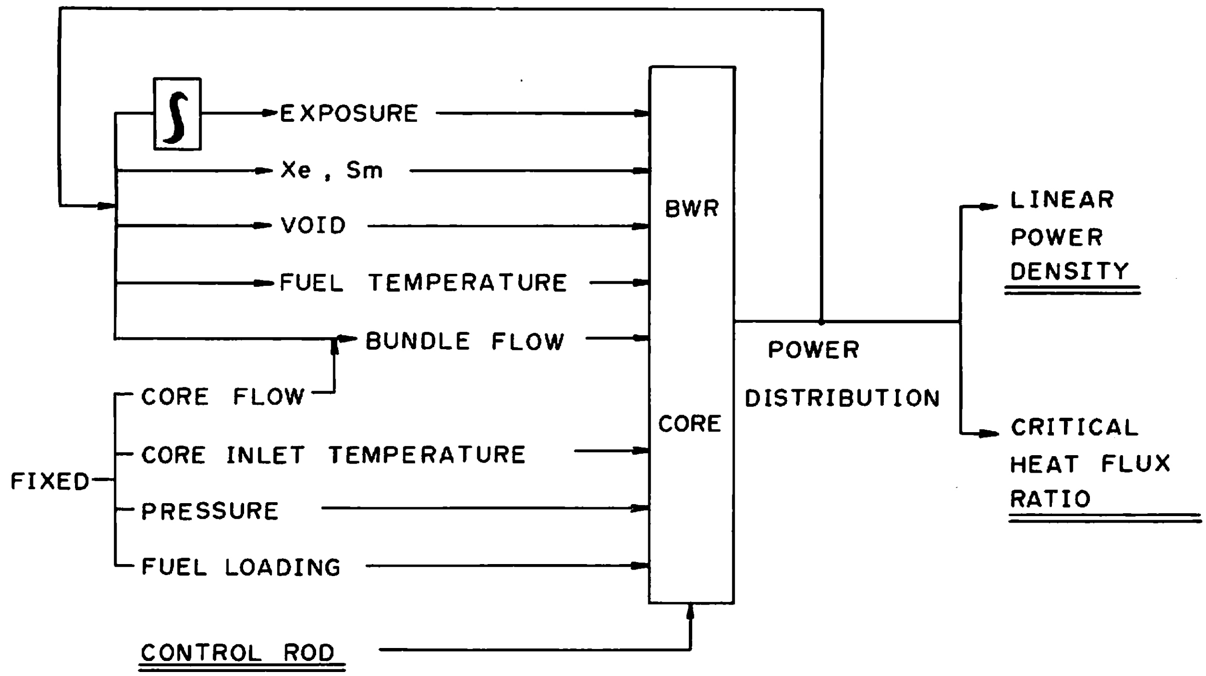


Fig. 3. Structure of 3D BWR core simulators.

pattern) in the reference reactor. It seems reasonable to postulate that the $\frac{1}{4}$ mirror symmetry is preserved during operation. With this in mind the number of rods in $\frac{1}{4}$ core reduces to 10 for both A- and B-patterns. The average exposure in one cycle is ~6 GWd/T, and the pattern change takes place at every 1 GWd/T interval. This requires about six different rod patterns per cycle. Thus, the number of unknown variables—depth of each rod for each pattern—is 60.

Two kinds of thermal constraints should be monitored at every segment. The number of segments is 1200:100 fuel bundles (for $\frac{1}{4}$ core) × 12 axial nodes (somewhat tentative). In addition, criticality at full power is another condition for each pattern.

The linear programming (LP) method is appropriate for the problem of such size.[13] The nonlinearity of the present problem can be treated by successive approximations. The method[14] is called MAP, method of approximate programming.

Decomposition into Two Heirarchies

In general, the computation time is not formidable for the nonlinear programming problem of the present size when MAP is used. The real problem lies in the complexity of the system equations; most of the computation time is spent on the renewal of LP coefficients. A small amount of certain control rod movement affects criticality and power distribution of the core at present and in the future. Such sensitivity data of a certain rod are generated by burnup calculations that require 6 to 1 (average 3.5) simulations. It is not practical to repeat the process for all 60 variables, and this is for only one step in the iteration process.

To relieve the difficulty, the space-time problem is decomposed into two hierarchies: outer loop problem and inner loop problem, as shown in Fig. 4. The former gives a target power distribution, Π_{ijk}, which does not change with time, i.e., to give a target burnup distribution at EOC. The latter problem searches for a rod pattern at each time step that best realizes the target power distribution:

$$\text{minimize} \quad J = \sum_{ijk} (P_{ijk} - \Pi_{ijk})^2 \quad . \tag{5}$$

The inner loop calculation repeats rod pattern determination and exposure addition step by step from the beginning to the end of cycle. At the end of cycle, the cycle length is evaluated, and the whole process is repeated with another target power distribution.

This decomposition assumes that target power distribution does not change with time. A better solution can be obtained by pursuing a time-varying target distribution. In this sense the solution is not optimal. However, this device is, in fact, essential in reducing computation time. The effect of the decomposition on the problem

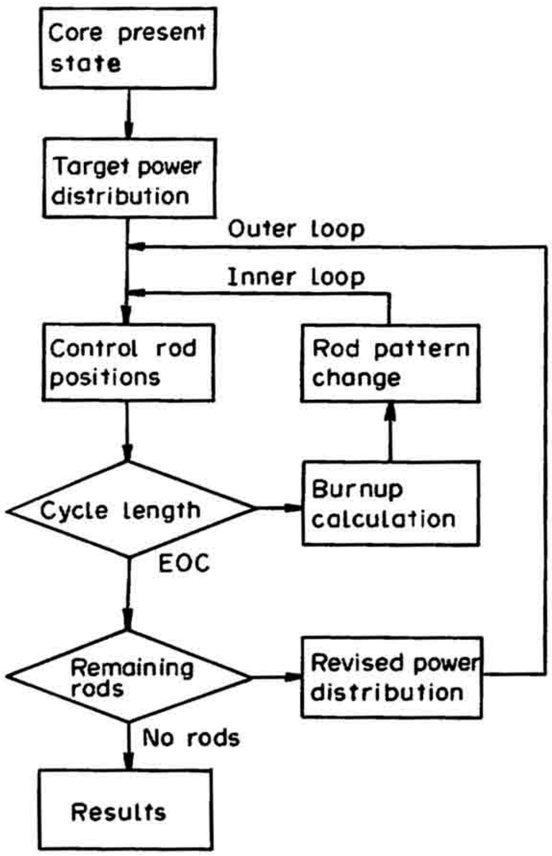


Fig. 4. Flow diagram of OPROD program.

TABLE II

Effect of the Decomposition on the Scale of Problem

Method	Original	Decomposed
Number of variables	60	10
Number of constraints		
criticality	6	1
linear power density	7200	1200
critical heat flux	7200	1200
Number of iterations	10[a]	5 × 3[b]
Number of 3D simulations	2100[c]	900[d]

[a]Guess.
[b]5 for inner, 3 for outer iteration.
[c]60 (variables) × 3.5 (burnup steps) × 10 (iterations).
[d]10 (variables) × 3.5 (burnup steps) × 15 (iterations).

size is shown in Table II. The time variable is eliminated in the inner loop problem, and one overall problem is reduced to six small problems of $\frac{1}{6}$ in size.

Target Power Distribution

There exists only one time-invariant power distribution that realizes complete withdrawal of control rods at the end of cycle. This is called the Haling distribution, and it is known to be the optimal distribution in the sense that the power peaking factor is minimum among rod programs that achieve "EOC with all rods out."[15] Because of this feature, the Haling principle has been adopted as a guide in BWR operations.

However, the principle cannot be practiced in the BWR for two reasons. One reason is of principle; the change in absorption density dictated by this principle is a continuous function of space that is approximately the same as power distribution. The shape of the control rod density cannot simulate the center-peaked power distribution because rods are inserted from the bottom. Another reason is technical; the BWR is not equipped with an appropriate facility to inform the operator as to which rod to maneuver to best fit the power distribution to a target distribution.

When the peaking factor of the Haling power distribution has a margin over the design value, it is possible to extend the cycle length beyond that obtained by the Haling principle. The two-region model concisely clarifies that the power distribution should be shifted to the bottom region as much as possible.[3,4] The extensive numerical search on the multiregion axial problem also gives a consistent result. To shift the power to the bottom, the control rods must be inserted in the upper region. However, this is neither advisable from peaking considerations nor mechanically possible. Thus the best that an operator can do for the downward shift of axial power distribution is uniform insertion. Actually this is often much too effective, and the peak in the lower part of the core has to be depressed by shallow control rods. Hence the empirical "deep and shallow principle" follows. In practice, half the rods in A- (or B-) pattern belong to deep and the rest to the shallow rods. The deep rods and shallow rods in the A1- (B1-) pattern are shallowly and deeply inserted, respectively, in the A2- (B2-) pattern.

As for the radial shape of the target distribution, no unique principle has been known except for extreme situations.[3] However, it is verified that the target exposure distribution obtained by Haling principle is near optimum.[8]

On the basis of the above information, the target power distribution is set as the product of Haling distribution and a modifying function. The modification can be made in various ways, and in the present paper, a linear function of axial node number k with a single parameter is adopted:

$$\Pi_{ijk} = [1 + C(\text{KMAX} + 1 - k)]H_{ijk} \quad , \quad k = 1, 2, \ldots, \text{KMAX} \quad , \qquad (6)$$

where H is the Haling power distribution and C, a skewing factor, is to be determined in the outer loop to maximize the reactivity life. Introduction of more shape parameters would improve the result at the expense of searching cost.

Core Simulator

Several BWR core simulators are known[16,17]; any of them can be used in the program because they have common input and output structures. Here, OPROD adopts a FLARE-type simulator. The code FLARE itself is well-documented and widely used. The accuracy of the code has been reported.[18] It depends on the choice of such fitting parameters as albedos and a mixing factor of transport kernels. Such parameters have been optimized using a core performance calculation.[19] Certain bias, as well as scatter, is observed in the discrepancy between the measurement and the model. Such discrepancy can be allowed for in setting the limit values FLPDMAX and CHFRMIN.

Equations

The input of the simulator is a set of rod depth $x_n (n = 1, 2, \ldots, N; N = 10$, for example). The output quantities are

$$y^m = f^m(x_1, x_2, \ldots, x_N) \quad , \qquad (7)$$

where y^m stands for λ, FLPD_{ijk}, CHFR_{ijk}, and J in Eqs. (1), (2), (3), and (5).

The number of equations for the thermal constraints at all nodes is 1200 each. However, the constraints need only be monitored at points where the thermal margin is small. Such monitoring points are recalculated at each rod configuration by the following procedure for each of FLPD and CHFR:

1. Find a point where FLPD is largest (CHFR smallest) for each bundle, and rank all the bundles in descending (ascending) order of FLPD (CHFR).
2. Select the bundle with rank 1.
3. Find the point of the maximum FLPD (minimum CHFR) of the selected bundle, and add one point with a certain axial distance on each side of the point selected (three points in all).
4. Add three more points around the second peak (bottom) if this exists in the same bundle, and if such a peak (bottom) is large (small) enough and the distance of the two peaks (bottoms) is large enough.
5. Select the next-ranked bundle. Skip the bundle if any of its four adjacent bundles has already been selected. This is to avoid local concentration of monitoring positions in the core.
6. Repeat steps 3, 4, and 5 until the number of monitoring points is equal to $Mc(=50$, for example).

The above-mentioned procedure does not guarantee that the peak point of the resulting power shape lies in the $Mc(50)$ monitoring points. The number of monitoring points is selected on the basis of the size of "the region of unity," in which the power change fraction is approximately common.[20] Note that thermal constraints are checked for all 1200 points when the 3D solution is obtained, and the final solution satisfies the constraints everywhere.

By changing the rod depth, x_n, one by one around a reference pattern independently, sensitivity data are obtained:

$$S_n^m = \Delta y^m / \Delta x_n \quad . \qquad (8)$$

Equations (1), (2), (3), and (5) are linearized to give the following equations for rod change, δx_n:

$$\lambda_{\text{target}} - \epsilon_0 \leq \sum_{n=1}^{N} S_n^{\lambda} \delta x_n + \lambda_0 \leq \lambda_{\text{target}} + \epsilon_0 \quad , \qquad (9)$$

$$\sum_{n=1}^{N} S_n^{Fm} \delta x_n + F_{m0} \leq \text{FLPDMAX} \quad , \quad m = 1, 2, \ldots, M_c \qquad (10)$$

$$\sum_{n=1}^{N} S_n^{Cm} \delta x_n + C_{m0} \geq \text{CHFRMIN} \quad , \quad m = 1, 2, \ldots, M_c \qquad (11)$$

$$\delta J = \sum_{n=1}^{N} S_n^J \delta x_n \quad , \qquad (12)$$

where λ_0, F_{m0}, and C_{m0} are the values at the reference pattern. The maximum movement of δx_n must be constrained to a certain value to avoid

1. infinite solution
2. violation of linearity approximation
3. rod tip going out of the core

and to represent stuck rods, if any. When δx_n is solved by LP, a reference rod pattern shifts to a new one:

$$x_n \leftarrow x_n + \delta x_n \quad . \qquad (13)$$

The procedure is repeated until two successive solutions satisfy the convergence criteria

$$-\epsilon_1 \leq \delta J \quad \text{or} \quad |\delta x_n| \leq \epsilon_2 \quad \text{for all } n \quad . \qquad (14)$$

The initial pattern need not satisfy all the constraints. This is a favorable feature, because some algorithms require a feasible solution as an initial guess.

Features to Save Computation Time

The following features have been incorporated into OPROD to save computation time and memory capacity:

1. Cycle length is evaluated by extrapolating the reactivity of the core (calculated in the no-rod hypothesis) versus burnup, instead of determining it by burnup calculations with subdivided exposure steps near the end of cycle.
2. Reduced axial node number; 12 seems to be the minimum sufficient, while 24 is a conventional practice.
3. The sensitivity data are most time-consuming to obtain. Therefore, the LP calculation is repeated for various constraint values of maximum allowable movement of δx_n, and the resulting pattern is put into the simulator [not in Eq. (12)]. This is to utilize a set of sensitivity data fully before going to the next rod pattern.

It is shown in the next section that the core life is insensitive to the skewing factor C if it gives a control rod program that leads to "EOC with all rods out." Thus, the outer loop of the present algorithm can be omitted by the proper input of C, at a slight sacrifice of optimality. The omission of the outer loop is a decisive factor for time saving.

RESULTS

Test calculations are performed on the reference core shown in Table I. Table III lists a set of test results with skewing factor as a parameter. Cycle length is shown in the unit of Haling life for convenience.

Reference Case

A series of the rod pattern for case 1 is illustrated in Fig. 5. Insertion depth for each rod is given for a $\frac{1}{4}$ core using a conventional unit. Figure 6 shows the average fraction of inserted rod depth as well as the reactivity that the reactor would have if all rods were withdrawn. The rod density is stationary at the exposure of 2 GWd/T, owing to the poison curtain depletion characteristics. The core life is longer than that obtained by the Haling principle. As stated by Haling,[15] the reactivity lifetime is extended at the expense of power peaking margin. The result clearly shows that the "deep and shallow principle" dominates; after a certain amount of exposure, all shallow rods are out and some of the deep rods turn to the shallow.

TABLE III

Summary of Results of the Control Rod Program Generated by the Present Method OPROD

OPROD				
No.	Skewing Factor	Cycle Length (Relative)	Min MCHFR	Max MFLPD
1	0.1	1.045	2.68	0.93
2	0.05	1.036	2.70	0.92
3	0	1.027	2.82	0.89
Haling Principle		1.000	3.37	0.74

Pattern change interval: 1.0 GWd/T.
Burnup calculation time step: 1.0 GWd/T.

The maximum of FLPD in the core (MFLPD), and the minimum of CHFR in the core (MCHFR), behave as shown in Figs. 7 and 8. They, of course, satisfy constraint as expected, and thermal margins are larger toward EOC contrary to our previous experience, which told us that there remained less margins as exposure proceeds. By the present method margins near EOC allow all rods withdrawn, because rods are not needed to maneuver power shapes. Rods can be withdrawn from the lower part of the core because this part has been depleted by the higher power density from the beginning, as seen in Fig. 9, and the depletion plays role of shallow rods.

Note that the average axial peaking factor of 1.65 in this figure is higher than the typical value (e.g., 1.57). This is because of the very flat radial power distribution of the solution. The limiting 3D peaking factor was set at 2.31 (total peaking factor 3.00/local peaking factor 1.30), which is imposed on the nodal power of the 3D simulator. The flat radial distribution contributed to axial power skewing and hence to longer core life. Figure 10 shows a histogram of bundle exposures obtained by Haling and OPROD. OPROD gives a larger flattened region in which bundles are uniformly irradiated. This is a favorable feature for uniform-scattered loading.

Effect of Target Power Distribution

Axial shape of target distribution is controlled by a skewing factor C. Figure 11 shows cycle

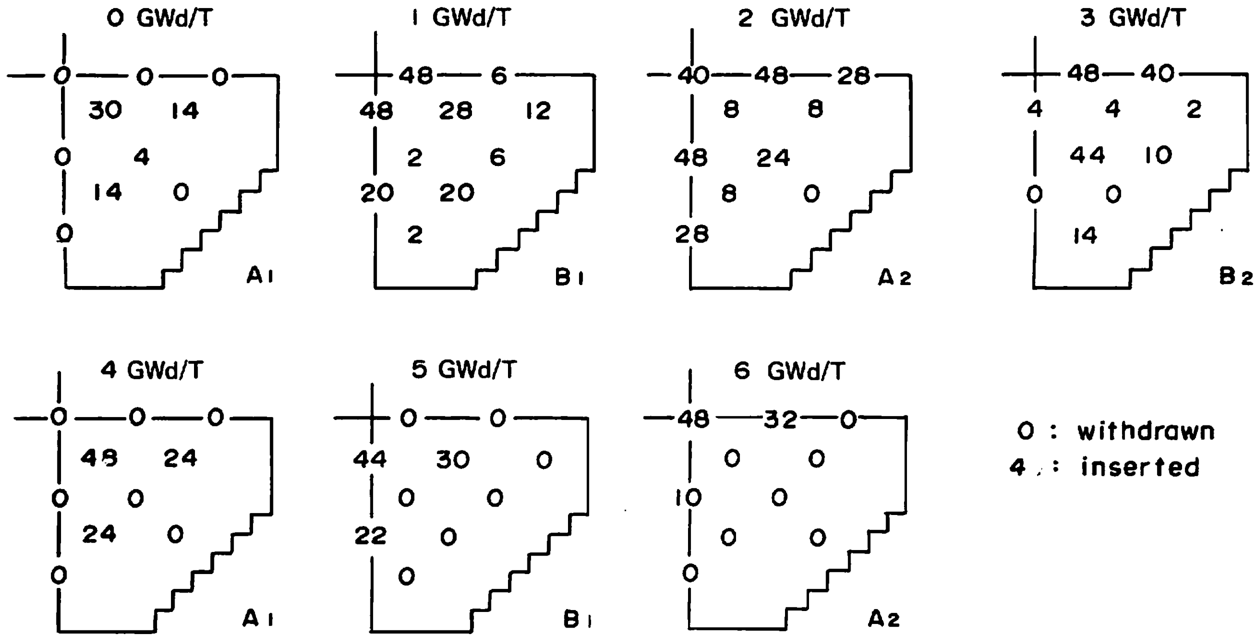


Fig. 5. Control rod program (case 1).

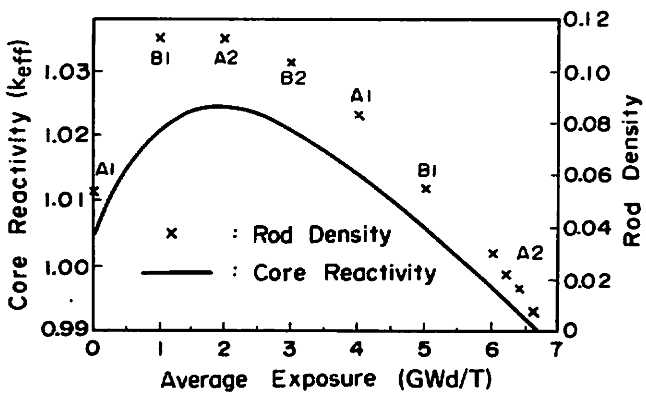


Fig. 6. Core reactivity (k_{eff} of core without control rods) and control rod density (case 1).

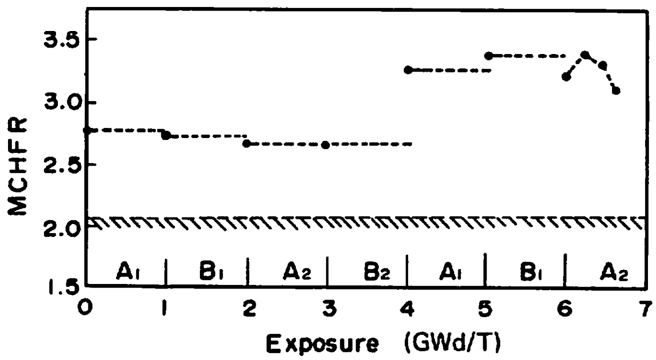


Fig. 8. Exposure versus minimum critical heat flux ratio as a result of control rod program by OPROD (case 1).

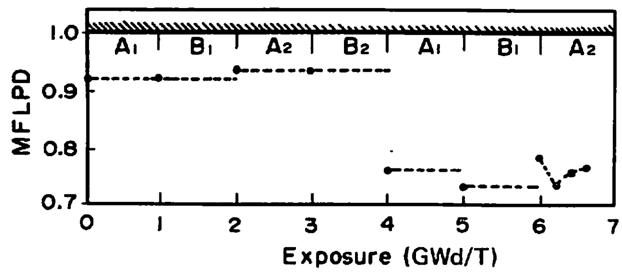


Fig. 7. Exposure versus maximum fraction of limiting power density as a result of control rod program by OPROD (case 1).

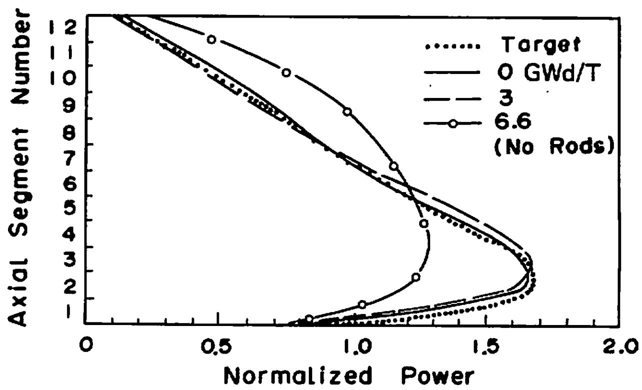


Fig. 9. Average axial power distribution versus exposure (case 1).

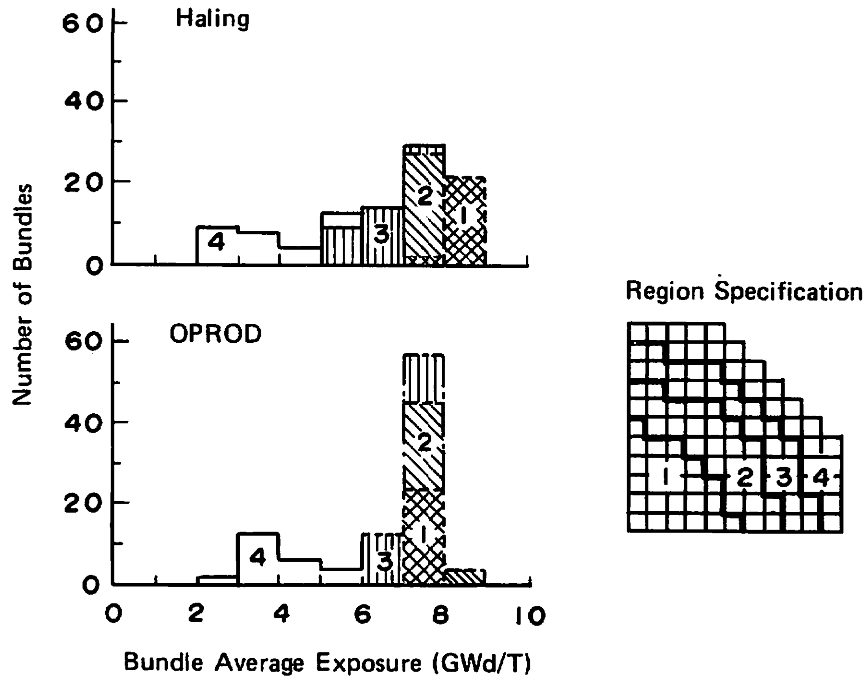


Fig. 10. EOC exposure distribution resulting from the operations by Haling principle and OPROD program.

length, maximum of MFLPD, and minimum of MCHFR through the core life as a function of the skewing factor *C*. As expected, more skewing (a larger value of *C*) yields conflicting effects: longer life and less thermal margin. The choice of *C* can be made by compromising the two conflicting factors. Greater skewing accentuates the deep and shallow split of rods, as seen in Fig. 12. It is shown that *C* = 0 gives a rod program that allows "EOC with all rods out." This means that unskewed Haling distribution itself is qualified as a target distribution in this test example.

Effect of Constraint

More stringent constraint on FLPD results in smaller values of MFLPD and cycle length. The relation is shown in Fig. 13. The skewing factor, *C*, is set to zero in these cases.

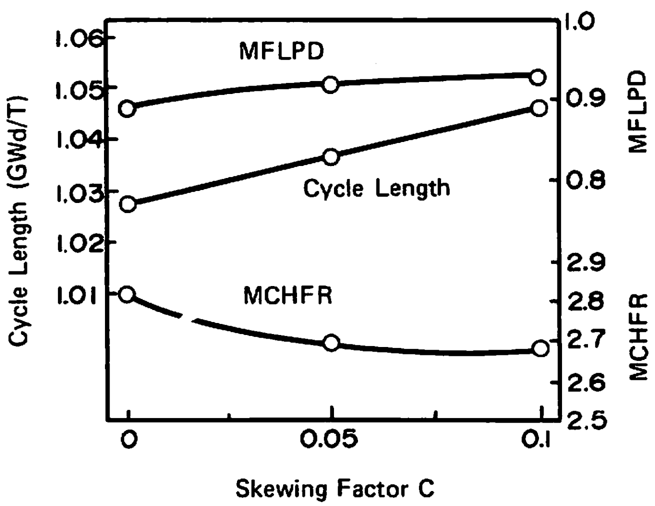


Fig. 11. Effect of the skewing factor on the cycle length, MFLPD, and MCHFR.

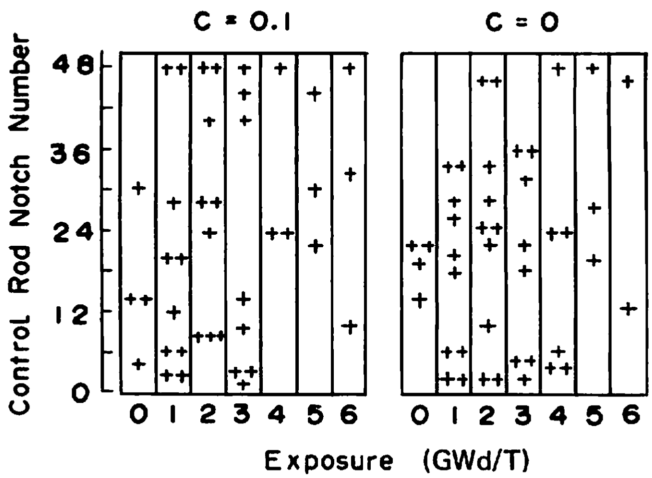


Fig. 12. Effect of the target power distribution on the rod position.

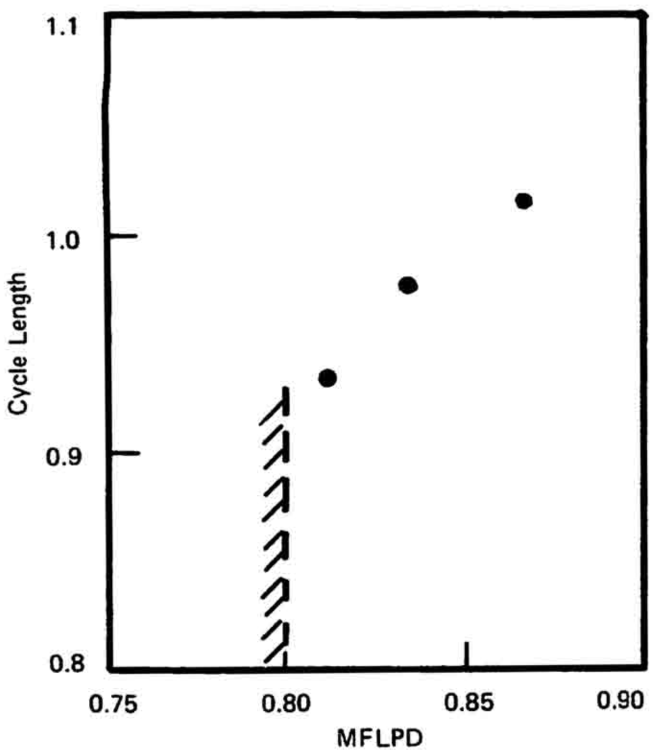


Fig. 13. Maximum fraction of limiting power density versus cycle length normalized by Haling's life.

Effect of Pattern Change Interval

Comparison of computations for different pattern change intervals, 0.5, 1.0, and 1.5 GWd/T, shows no meaningful trends in cycle length and thermal margins in a specific test example.

DISCUSSION AND CONCLUSION

A practical algorithm, OPROD, for generation of a control rod program is described. The algorithm consists of a combination of the linear programming and a 3D core simulator. The barrier of memory capacity and computation time has been overcome by utilizing information accumulated in recent years by operational experiences and theoretical analyses. Decomposition of the problem into two loops and selection of monitoring points for thermal constraints are main items that have contributed to reduce the problem to a manageable size.

The size of the memory required for this code is shown in Table IV, assuming a quarter-core analysis. The computing time is nearly proportional to the number of the 3D core simulation, shown in Table II.

TABLE IV

Memory Capacity Required for OPROD

Electric Output [MW(e)]	Number of Bundles	Memory (1000 words)	
		Main	Disk
460	400	86	170
780	548	94	240
1100	764	105	330

The test computation has demonstrated the practicability of the algorithm. It also has shown some characteristics in the in-core management problems, such as the effect of the target power distribution on the reactivity life.

Several items for further developments have been found on OPROD:

1. Computation time has still room for reduction. More than 70% of the time is spent in 3D simulations. Response of segment power to small changes of control rod position should be obtained by an improved method.

2. Grouping of control rods will contribute not only to save computation time but also to express control method more concisely. The optimum number of degrees of freedom is a question left in the future.

3. The accuracy of the core model is a perpetual problem. An improved model can and should be adopted within the framework of the present algorithm.

4. The cost function J has two bottoms against rod position, x_n, and thus a proper initial guess and a means that allows global search are desirable.

The present computer code, OPROD, has achieved the following:

1. The code has dispensed with the manual search and has facilitated prompt responses to unexpected changes of operating conditions.

2. Observation of the thermal limits and suboptimization of the cycle length are achieved more easily by OPROD.

3. The code can also be applied for the problem of lowering the maximum of linear power density. This is readily achieved by imposing a stricter constraint on the linear

power density. It contributes to safe operation of BWRs, because the linear power density is an important factor in preventing fuel failures.

Extensive application of the present method will contribute to the safety and economy of BWR operation.

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