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Multiregion Neutronics Model Based on Coarse Mesh Nodal Coupling Method for Transient Analyses of Boiling Water Reactors

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A coarse mesh nodal coupling method, a well-known technique often used in steady-state neutronics analysis of light water reactors, is extended to a problem of transient phenomena of boiling water reactors (BWRs). Spatial collapse is attempted to develop a multiregion neutronics model and the associated axially one-dimensional and one-point models.

These models are numerically solved through the use of two approximations, quasi-static and prompt jump. The results as applied to a reference BWR core for transient analyses, initiated by artificial thermal-hydraulic disturbances, are presented to show the practicality of the approach.

The nature of the optimal weighting function necessary for the spatial collapse and for the quasi-static approximation is also discussed.

I. INTRODUCTION

Simulation techniques of transient phenomena are essential for boiling water reactor (BWR) core design and plant control. Transient analysis is more difficult than steady-state analysis both in terms of modeling and numerical computation.

The BWR core is composed of hundreds of fuel bundles. Each fuel bundle forms a channel along which coolant flows. Since there is no cross-flow among channels, a one-dimensional separated flow model[1] is believed to be good enough to describe the two-phase flow phenomena in a channel. However, these bundles are neutronically coupled with each other. Therefore, the neutronics model must take into account reactivity feedbacks in such a way that spatial variations of disturbances (fuel temperatures and moderator densities) are appropriately treated.

A three-dimensional treatment of the neutronics would be the most straightforward approach. However, this is neither practical nor economical. Some simplification is desired.

The BWR is characterized by the existence of an axial void distribution, and it is important to evaluate its change along the axial direction during transients. Therefore, a multinode treatment of the axial direction would be a minimum requirement.

The degree of sophistication of the radial direction treatment depends on the phenomena to be analyzed. A radially collapsed, axial one-dimensional model would be sufficient for the analysis of a turbine trip accident in which core characteristics are considered to behave uniformly in the radial direction. However, at least a three-dimensional equivalent model would be required for analyses of such accidents as one recirculation pump trip and an anticipated transient without scram with some control rods not inserted because spatial distortion of the neutron flux plays an important role in these cases.

[1]G. B. WALLIS, *One Dimensional Two-Phase Flow*, McGraw-Hill Book Company, New York (1969).

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A conventional fine mesh numerical method is limited by computing time and memory even for steady-state calculations.[2] The MEKIN code,[3] which solves three-dimensional power distribution on the basis of a few group time-dependent fine mesh diffusion method, is recognized as one of the most sophisticated kinetics codes.

A coarse mesh nodal coupling approach is a natural way to improve the computational efficiency. Attempts have been made to improve the accuracy without sacrificing the efficiency. Finnemann and Raum[4] introduced the nodal expansion method (NEM), which uses polynomial expansions of the fluxes and currents to determine the flux-current coupling terms. Smith[5] extended the analytical nodal method, similar to NEM, to three-dimensional geometry and showed that the computational efficiency can be at least two orders of magnitude greater than that of the conventional finite difference method.

It is felt, however, that much more efficiency needs to be attained for the method to be applicable to design calculations. The method would be practical if the time required in a neutronics model for simulating core behavior is within 20 times of real time by a standard large computer, e.g., an IBM 3033.

The FLARE model,[6] widely used as a steady-state BWR simulator, employs crude approximation but has some adjustable parameters such as albedos and mixing kernels. With these parameters properly chosen, it simulates core performance fairly satisfactorily.[7] Furthermore, this model can be applied to core management purposes that require hundreds of three-dimensional power distribution calculations to optimize operating strategy because of its efficiency.[8,9] The FLARE-type nodal coupling method also provides effective core-averaged properties for use in the point kinetics equations.[10]

Extension of the FLARE model to dynamics problems for transients with time constants larger than the neutron slowing down and diffusion time ($\sim 2 \times 10^{-4}$ s) is straightforward. However, it is still time consuming and impractical even by this model to solve transient behavior of three-dimensional nodal power distribution. Some spatial collapse over the radial direction is required for the method to be practical.

The aim of this paper is to develop a multiregion neutronics model based on a coarse mesh nodal coupling method (FLARE) for transient analyses of BWRs (Ref. 11). Derivation of equations, method of numerical solutions, and application results to a commercial BWR are given.

II. NEUTRONICS MODEL BASED ON COARSE MESH NODAL COUPLING METHOD

II.A. Steady-State Equation and Importance

The steady-state neutron balance equation of the FLARE model is given by

$$S_i = \frac{k_{\infty i}}{\lambda}\left(\sum_j W_{ji} S_j + WS_i S_i\right) , \qquad (1)$$

where

S_i = neutron source, number of neutrons produced per node i

W_{ji} = neutron transport kernel, probability that a neutron born at node j is finally absorbed at node i

WS_i = self-transport kernel, probability that a neutron born at node i is finally absorbed at the same node i

$k_{\infty i}$ = infinite neutron multiplication factor at node i

λ = eigenvalue, effective neutron multiplication factor.

The self-transport kernel WS_i is related to the neutron transport kernel by

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[2]M. R. WAGNER, "Current Trend in Multi Dimensional Static Reactor Calculations," *Proc. Conf. Computing Methods in Nuclear Engineering*, Charleston, South Carolina, April 15-17, 1975, CONF-750413, **1**, 1, U.S. Energy Research and Development Administration (1975).

[3]R. W. BOWRING et al., "MEKIN: MIT-EPRI Nuclear Reactor Kinetics Code," EPRI CCM-1, Electric Power Research Institute (1975).

[4]H. FINNEMANN and H. RAUM, "Nodal Expansion Method for the Analysis of Space-Time Effects in LWRs," *Proc. NEACRP Specialists' Mtg. Calculation of Three-Dimensional Rating Distribution in Operating Reactors*, Paris, France, November 1979, Nuclear Energy Agency Committee on Reactor Physics (1980).

[5]K. S. SMITH, "An Analytic Nodal Method for Solving the Two-Group, Multidimensional, Static and Transient Neutron Diffusion Equations," Thesis, Massachusetts Institute of Technology (Mar. 1979).

[6]D. L. DELP et al., "FLARE: A Three-Dimensional Boiling Water Reactor Simulator," GEAP-4598, General Electric Company (1964).

[7]T. KIGUCHI and T. KAWAI, *Nucl. Technol.*, **27**, 315 (1975).

[8]T. HAYASE and H. MOTODA, *Nucl. Technol.*, **48**, 91 (1980).

[9]Y. BESSHO et al., *J. Nucl. Sci. Technol.*, **18**, 697 (1981).

[10]J. K. PARK and M. BECKER, *Trans. Am. Nucl. Soc.*, **35**, 581 (1980).

[11]H. MOTODA et al., *Trans. Am. Nucl. Soc.*, **38**, 352 (1981).

$$WS_i = 1 - \sum_j{}' W_{ij} - \sum_j{}'' (1 - \alpha_{ij}) W_{ij} \ , \qquad (2)$$

where

α_{ij} = albedo, reflection rate of a neutron going to node j from node i (defined as zero for nodes having six existing neighbors)

$\sum_j{}'$ = summation over existing neighbor node j

$\sum_j{}''$ = summation over missing neighbor node j.

The eigenvalue λ can be obtained by taking a summation over all nodes i in Eq. (1) as

$$\lambda = \frac{\sum_i \left(\sum_j W_{ji} S_j + WS_i S_i \right)}{\sum_i \frac{S_i}{k_{\infty i}}}$$

$$= \frac{\sum_i S_i - \sum_i S_i \sum_j{}'' (1 - \alpha_{ij}) W_{ij}}{\sum_i \frac{S_i}{k_{\infty i}}} \ . \qquad (3)$$

The physical meaning of Eq. (3) is the ratio (production-leakage)/absorption.

The equation adjoint to Eq. (1) is given by

$$\tilde{S}_i = \frac{1}{\lambda} \left(\sum_j k_{\infty j} W_{ij} \tilde{S}_j + k_{\infty i} WS_i \tilde{S}_i \right) \ . \qquad (4)$$

The variable $\tilde{S}_i$ physically means the importance of a neutron produced and is hereafter designated source importance. Note that the external boundary condition is automatically incorporated into Eq. (4). This is evident by interpreting Eq. (4) as an importance balance.

Importance is phenomena dependent and is not uniquely required. Importance of an absorbed neutron can be conceivable. The equation that this importance, designated absorption importance, satisfies is given by

$$S_i^* = \frac{k_{\infty i}}{\lambda} \left(\sum_j W_{ij} S_j^* + WS_i S_i^* \right) \ . \qquad (5)$$

Equation (5) is shown to be adjoint to Eq. (6), the balance of absorption rate A_i:

$$A_i = \frac{1}{\lambda} \left(\sum_j k_{\infty j} W_{ji} A_j + k_{\infty i} WS_i A_i \right) \ . \qquad (6)$$

The following relation holds for the above variables:

$$S_i \tilde{S}_i = A_i S_i^* \ , \qquad (7)$$

and the eigenvalues of Eqs. (1), (4), (5), and (6) are the same.

The two kinds of importance can be derived from a different viewpoint. The eigenvalue λ of Eq. (1) can also be obtained in two other ways introducing a weighting function ω_i as

$$\lambda = \frac{\sum_i \omega_i \left(\sum_j W_{ji} S_j + WS_i S_i \right)}{\sum_i \frac{\omega_i S_i}{k_{\infty i}}} \qquad (8)$$

and

$$\lambda = \frac{\sum_i \omega_i k_{\infty i} \left(\sum_j W_{ji} S_j + WS_i \right)}{\sum_i \omega_i S_i} \ . \qquad (9)$$

These eigenvalues are the same if S_i is known exactly. It is possible to obtain λ, correct to the first order, if the weight ω_i is chosen such that Eqs. (8) and (9) are stationary for any first-order variation of S_i. The variational principle gives

$$\delta\lambda = \sum_i \frac{\partial \lambda}{\partial S_i} \delta S_i = 0 \rightarrow \frac{\partial \lambda}{\partial S_i} = 0 \ , \qquad (10)$$

and it is shown that $\omega_i = S_i^*$ for Eq. (8) and $\omega_i = \tilde{S}_i$ for Eq. (9). In other words, these two kinds of importance have the meaning of the optimal weighting function.

II.B. Space-Dependent Dynamics Equation

Equation (1) can be rewritten as

$$\sum_j W_{ji} S_j + WS_i S_i - \lambda \frac{S_i}{k_{\infty i}} = 0 \ . \qquad (11)$$

The left side of Eq. (11) is decomposed into three terms—net inflow, production, and absorption:

$$\underbrace{\left[\sum_j W_{ji} S_j - \sum_j{}' W_{ij} S_i - \sum_j{}'' (1 - \alpha_{ij}) W_{ij} \right]}_{\text{(net inflow)}} + \underbrace{S_i}_{\text{(production)}} \underbrace{- \lambda \frac{S_i}{k_{\infty i}}}_{\text{(absorption)}} = 0 \ . \qquad (12)$$

The first term describes the net number of neutrons coming into the node i and absorbed there. However, this term can be thought to represent the net inflow. This is because the neutrons that come into a node and go out of it to a neighboring node without being absorbed are balanced by the neutrons that take the same process in other nodes unless the time scale of interest is in the range

of the neutrons slowing down and diffusion time ($\sim 2 \times 10^{-4}$ s).

Taking a dynamic balance of neutrons, the following kinetic equation is obtained:

$$l_i \frac{\partial S_i}{\partial t} = \sum_j W_{ji} S_j + \left(W S_i - \frac{\lambda_0}{k_{\infty i}} - \beta_i \right) S_i + \sum_l \lambda_l C_{li} \ ,$$

$$\frac{\partial C_{li}}{\partial t} = \beta_{li} S_i - \lambda_l C_{li} \ ,$$

and

$$P_i = \frac{S_i}{\nu_i} \ , \tag{13}$$

where

l_i = neutron generation time at node i

β_{li} = l'th group delayed neutron precursor fraction at node i ($l = 1, 2, \ldots, L$),

$$\beta_i = \sum_l \beta_{li}$$

λ_l = l'th group delayed neutron precursor decay constant

C_{li} = l'th group delayed neutron precursor concentration at node i

P_i = power at node i

ν_i = number of neutrons emitted per fission at node i

λ_0 = eigenvalue at initial steady state.

In deriving Eq. (13), time variation of l_i is ignored, but l_i is allowed to vary with time in the equation. Equation (13) is the three-dimensional kinetic equation based on a coarse mesh nodal coupling method.

Use of one-group approximation implies an instantaneous change in neutron spectrum. Since the time constant for spectrum change is estimated to be $\sim 10^{-4}$ s, about the same for the time required for neutron slowing down and diffusion, Eq. (13) is thought to be valid for transients with a time constant larger than $\sim 2 \times 10^{-4}$ s.

III. METHOD OF SPATIAL COLLAPSE

III.A. Multiregion Neutronics Model

A reactor core is divided radially into several regions of arbitrary sizes. The concept of region specification is shown in Fig. 1. A region can be a set of separate subregions. A region number is denoted as q ($q = 1, 2, \ldots, Q$). Axial nodal division is the same as the original node i, and the axial node is denoted as k ($k = 1, 2, \ldots, K$).

Let S_i and C_{li} be factored into two functions as

$$S_i = \hat{n}_{q,k} \hat{\psi}_i$$

and

$$C_{li} = m_{lq,k} K_{li} \ , \tag{14}$$

where $\hat{n}_{q,k}$ and $m_{lq,k}$ are new variables that are assumed to be constant within region q for axial node k, representing regionwise behavior. The variables ψ_i and K_{li} are considered to be correction functions, and their changes in space and time are assumed to be small.

The following normalization is imposed to make this split unique:

$$\hat{n}_{q,k} = \langle \omega_i S_i \rangle_{i \epsilon q,k}$$

and

$$m_{lq,k} = \langle \omega_{li} C_{li} \rangle_{i \epsilon q,k} \ . \tag{15}$$

Here, the variables ω_i and ω_{li} are yet unknown weights and $\langle \ \rangle_{i \epsilon q,k}$ designates the summation over all nodes i that are in region q at axial node k. From Eqs. (14) and (15), the following constraints are derived:

$$\langle \omega_i \hat{\psi}_i \rangle_{i \epsilon q,k} = 1.0$$

and

$$\langle \omega_{li} K_{li} \rangle_{i \epsilon q,k} = 1.0 \ . \tag{16}$$

Furthermore, introducing a new variable $n_{q,k}$ defined as

$$n_{q,k}(t) = \frac{\hat{n}_{q,k}(t)}{\hat{n}_{q,k}(0)} = \frac{\hat{n}_{q,k}(t)\hat{\psi}_i}{\hat{n}_{q,k}(0)\hat{\psi}_i} \simeq \frac{S_i(t)}{S_i(0)} \ , \tag{17}$$

the following initial conditions are derived:

$$n_{q,k}(0) = 1.0 \ , \quad \hat{\psi}_i(0) = \frac{S_i(0)}{\hat{n}_{q,k}(0)} \ ,$$

$$m_{lq,k}(0) = \frac{1}{\lambda_l} \langle \omega_i \beta_{li} S_i(0) \rangle_{i \epsilon q,k} \ ,$$

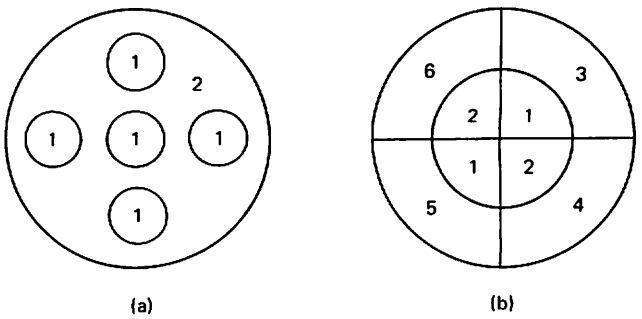

Fig. 1. Concept of region specification: (a) two-region specification and (b) six-region specification.

and

$$K_{li}(0) = \frac{\beta_{li} S_i(0)}{\langle \omega_{li} \beta_{li} S_i(0) \rangle_{i \epsilon q,k}} \quad . \tag{18}$$

To determine the optimal weights ω_i and ω_{li}:

1. Equation (14) is substituted into S_i and C_{li} in Eq. (13).
2. The first equation is multiplied by ω_i and the second by ω_{li}.
3. These are integrated over all nodes as

$$\sum_{q,k} [\dot{n}_{q,k} \langle \omega_i l_i \hat{\psi}_i \rangle_{i \epsilon q,k} \hat{n}_{q,k}(0)] + \sum_{q,k} [n_{q,k} \langle \omega_i l_i \dot{\hat{\psi}}_i \rangle_{i \epsilon q,k} \hat{n}_{q,k}(0)] = \sum_{q,k} \left\langle \omega_i \sum_j W_{ji} n_{q'',k} \hat{n}_{q'',k}(0) \hat{\psi}_j \right\rangle_{i \epsilon q,k} + \sum_{q,k} \left[n_{q,k} \left\langle \omega_i \left(W S_i - \frac{\lambda_0}{k_{\infty i}} - \beta_i \right) \hat{\psi}_i \right\rangle_{i \epsilon q,k} \hat{n}_{q,k}(0) \right] + \sum_{q,k} \sum_l (\lambda_l m_{lq,k} \langle \omega_i K_{li} \rangle_{i \epsilon q,k})$$

and

$$\sum_{q,k} (\dot{m}_{lq,k} \langle \omega_{li} K_{li} \rangle_{i \epsilon q,k}) + \sum_{q,k} (m_{lq,k} \langle \omega_{li} \dot{K}_{li} \rangle_{i \epsilon q,k}) = \sum_{q,k} [n_{q,k} \langle \omega_{li} \beta_{li} \hat{\psi}_i \rangle_{i \epsilon q,k} \hat{n}_{q,k}(0)] - \sum_{q,k} (\lambda_l m_{lq,k} \langle \omega_{li} K_{li} \rangle_{i \epsilon q,k}) \quad , \tag{19}$$

where q'' means a region number in which the node j belongs.

Since an approximation is made that $\hat{\psi}_i$ and K_{li} remain unchanged in the multiregion model, the optimal weights ω_i and ω_{li} can be determined by the condition that Eq. (19) is stationary for any first-order variation of $\hat{\psi}_i$ and K_{li}: $\delta \hat{\psi}_i$ and δK_{li}.

The first-order perturbation of Eq. (19) with respect to $\hat{\psi}_i$ and K_{li} leads to

$$\sum_{q,k} [\dot{n}_{q,k} \langle \omega_i l_i \delta \hat{\psi}_i \rangle_{i \epsilon q,k} \hat{n}_{q,k}(0)] + \sum_{q,k} [n_{q,k} \langle \omega_i l_i \delta \dot{\hat{\psi}}_i \rangle_{i \epsilon q,k} \hat{n}_{q,k}(0)] = \sum_{q,k} \left\langle \omega_i \sum_j W_{ji} n_{q'',k} \hat{n}_{q'',k}(0) \, \delta \hat{\psi}_j \right\rangle_{i \epsilon q,k} + \sum_{q,k} \left[n_{q,k} \left\langle \omega_i \left(W S_i - \frac{\lambda_0}{k_{\infty i}} - \beta_i \right) \delta \hat{\psi}_i \right\rangle_{i \epsilon q,k} \hat{n}_{q,k}(0) \right] + \sum_{q,k} \sum_l (\lambda_l m_{lq,k} \langle \omega_i \delta K_{li} \rangle_{i \epsilon q,k}) \tag{20}$$

and

$$\sum_{q,k} [\dot{m}_{lq,k} \langle \omega_{li} \delta K_{li} \rangle_{i \epsilon q,k}] + \sum_{q,k} (m_{lq,k} \langle \omega_{li} \delta \dot{K}_{li} \rangle_{i \epsilon q,k}) = \sum_{q,k} [n_{q,k} \langle \omega_{li} \beta_{li} \delta \hat{\psi}_i \rangle_{i \epsilon q,k} \hat{n}_{q,k}(0)] - \sum_{q,k} (\lambda_l m_{lq,k} \langle \omega_{li} \delta K_{li} \rangle_{i \epsilon q,k}) \quad . \tag{21}$$

From Eq. (16), we obtain

$$\langle \omega_i \delta \hat{\psi}_i \rangle_{i \epsilon q,k} = 0$$

and

$$\langle \omega_{li} \delta K_{li} \rangle_{i \epsilon q,k} = 0 \quad . \tag{22}$$

Using Eq. (22), the following relations are derived:

$$\langle \omega_i l_i \delta \hat{\psi}_i \rangle_{i \epsilon q,k} = 0 \quad ,$$

$$\langle \omega_i l_i \delta \dot{\hat{\psi}}_i \rangle_{i \epsilon q,k} = -\left\langle \frac{\partial \omega_i}{\partial t} \, l_i \delta \hat{\psi}_i \right\rangle_{i \epsilon q,k} \quad ,$$

and

$$\langle \omega_{li} \delta \dot{K}_{li} \rangle_{i \epsilon q,k} = -\left\langle \frac{\partial \omega_{li}}{\partial t} \, \delta K_{li} \right\rangle_{i \epsilon q,k} \quad . \tag{23}$$

Here, l_i is assumed to be nearly constant in space and time. The assumption of spatial uniformity of l_i within a region q for a specified height k is justified because a region is a set of nodes having similar nuclear properties. The assumption of $\dot{l}_i \sim 0$ may not hold for some transients because l_i is sensitive to spectrum change, e.g., $\Delta l/l \sim 0.4\%$ for a 1% change of void. This assumption, however, is used only for deriving the optimal weights.

The following equation is obtained by adding Eqs. (20) and (21) after using the above relations and exchanging i and j of the first term of the right side of Eq. (20):

$$\sum_{q,k} \left\{ n_{q,k} \left\langle \left[l_i \frac{\partial \omega_i}{\partial t} + \sum_j W_{ij} \omega_j + \left(W S_i - \frac{\lambda_0}{k_{\infty i}} - \beta_i \right) \omega_i + \sum_l \beta_{li} \omega_{li} \right] \delta \hat{\psi}_i \right\rangle_{i \epsilon q,k} \hat{n}_{q,k}(0) \right\} + \sum_{q,k} \sum_l \left\langle \left(\frac{\partial \omega_i}{\partial t} + \lambda_l \omega_i - \lambda_l \omega l_i \right) \delta K_{li} \right\rangle_{i \epsilon q,k} = 0 \quad . \tag{24}$$

Here, $\langle \omega_{li} \delta K_{li} \rangle_{i \epsilon q,k}$ is retained in the second term since this has no influence on Eq. (24) because of the condition in Eq. (22). The stationary condition, therefore, leads to

$$-l_i \frac{\partial \omega_i}{\partial t} = \sum_j W_{ij} \omega_j + \left(W S_i - \frac{\lambda_0}{k_{\infty i}} - \beta_i \right) \omega_i + \sum_l \beta_{li} \omega_{li}$$

and

$$-\frac{\partial \omega_{li}}{\partial t} = \lambda_l \omega_i - \lambda_l \omega_{li} \quad . \tag{25}$$

Since the time variations of shape $\hat{\psi}_i$ and K_{li} are assumed to be small, it is reasonable to approximate the solutions of Eq. (25) by the initial state values:

$$\sum_j W_{ij}\omega_j + \left(WS_i - \frac{\lambda_0}{k_{\infty i}}\right)\omega_i = 0$$

and

$$\omega_{li} = \omega_i \quad . \tag{26}$$

It is interesting to note that the optimal weights for $\hat{\psi}_i$ and K_{li} are the same under this approximation and are equal to the absorption importance defined in Sec. II.A.

It should be mentioned that this stationary expression could have been obtained directly from Eq. (13) by assuming stationality of S_i and C_{li}. (Note that this assumption is only for obtaining the optimal weights.)

The equations that $n_{q,k}$ and $m_{lq,k}$ satisfy can be obtained by taking a summation over only i values within q and k (not over all nodes) in Eq. (19):

$$\langle S_i^* l_i S_i^0\rangle_{i\epsilon q,k}\dot{n}_{q,k} = \left\langle S_i^* \sum_j W_{ji} S_j^0 n_{q'',k}\right\rangle_{i\epsilon q,k} + \left(\langle S_i^* WS_i S_i^0\rangle_{i\epsilon q,k} - \left\langle S_i^* \frac{\lambda_0}{k_{\infty i}} S_i^0\right\rangle_{i\epsilon q,k} - \langle S_i^* \beta_i S_i^0\rangle_{i\epsilon q,k}\right) n_{q,k} + \sum_l \lambda_l m_{lq,k}$$

and

$$\dot{m}_{lq,k} = \langle S_i^* \beta_{li} S_i^0\rangle_{i\epsilon q,k} n_{q,k} - \lambda_l m_{lq,k} \quad . \tag{27}$$

Here, S_i^* and S_i are substituted in ω_i, ω_{li} and $\hat{n}_{q,k}(0)\hat{\psi}_i$, respectively, and Eq. (16) is used.

The summation over j of the first term of the right side of the first equation of Eq. (27) can be split into four terms:

$$\left\langle S_i^* \sum_j W_{ji} S_j^0 n_{q'',k}\right\rangle_{i\epsilon q,k} = \langle S_j^* W_{ij} S_i^0\rangle_{i,j\epsilon q,k} n_{q,k} + \langle S_i^* W_{ji} S_j^0\rangle_{\substack{i\epsilon q,k \\ j\epsilon q,k}} + \langle S_i^* W_{i_{k+1}i} S_{i_{k+1}}^0\rangle_{i\epsilon q,k} n_{q,k+1} + \langle S_i^* W_{i_{k-1}i} S_{i_{k-1}}^0\rangle_{i\epsilon q,k} n_{q,k-1} \quad , \tag{28}$$

where $j \in q'',k$ is divided into two sets—$j \in q,k$ (inside region q) and $j \in q',k$ (outside region q), and where i_{k+1} means the node directly above i and i_{k-1} the node directly below i.

Final equations for $n_{q,k}$ and $m_{lq,k}$ are written as

$$l_{q,k}\dot{n}_{q,k} = \sum_{q'} WH_{q',q,k}\, n_{q',k} + W^-_{q,k+1} n_{q,k+1} + W^+_{q,k-1} n_{q,k-1} + \left(W_{q,k} + WS_{q,k} - \frac{\lambda_0}{k_{\infty q,k}} - \beta_{q,k}\right) n_{q,k} + \sum_l \lambda_l m_{lq,k}$$

and

$$\dot{m}_{lq,k} = \beta_{lq,k} n_{q,k} - \lambda_l m_{lq,k} \quad , \tag{29}$$

with the initial conditions

$$n_{q,k}(0) = 1.0$$

and

$$m_{lq,k}(0) = \frac{\beta_{lq,k}}{\lambda_l} \quad . \tag{30}$$

The coefficients are defined as

$$l_{q,k} = \sum_{i\epsilon q,k} S_i^* l_i S_i^0 \quad ,$$

$$WH_{q',q,k} = \sum_{i\epsilon q,k} \sum_{j\epsilon q',k} S_i^* W_{ji} S_j^0 \quad ,$$

$$W^-_{q,k} = \sum_{i\epsilon q,k} S^*_{i_{k-1}} W_{ii_{k-1}} S_i^0 \quad ,$$

$$W^+_{q,k} = \sum_{i\epsilon q,k} S^*_{i_{k+1}} W_{ii_{k+1}} S_i^0 \quad ,$$

$$W_{q,k} = \sum_{i\epsilon q,k} \sum_{j\epsilon q,k} S_j^* W_{ij} S_i^0 \quad ,$$

$$WS_{q,k} = \sum_{i\epsilon q,k} S_i^* WS_i S_i^0 \quad ,$$

$$k_{\infty q,k} = 1 \Big/ \sum_{i\epsilon q,k} (S_i^* S_i^0 / k_{\infty i}) \quad ,$$

$$\beta_{lq,k} = \sum_{i\epsilon q,k} S_i^* \beta_{li} S_i^0 \quad ,$$

and

$$\beta_{q,k} = \sum_l \beta_{lq,k} \quad . \tag{31}$$

Equation (29) is the multiregion kinetics equation based on a coarse mesh nodal coupling method. The power $P_{q,k}$ is obtained by Eq. (32) as

$$P_{q,k} = P_{q,k}(0) n_{q,k} \quad , \tag{32}$$

where

$$P_{q,k}(0) = \frac{\sum_{i\epsilon q,k} P_i(0)}{\sum_{i\epsilon q,k}} \quad , \quad \left[\frac{\sum_i P_i(0)}{\sum_i} = 1.0\right] . \tag{33}$$

III.B. Axial One-Dimensional Neutronics Model

Axial one-dimensional approximation requires integration of the neutron source in the radial direction. Taking summation over q in Eq. (29), the following equations are obtained:

$$l_k \dot{n}_k = W^-_{k+1} n_{k+1} + W^+_{k-1} n_{k-1} + \left(W_k + WS_k - \frac{\lambda_0}{k_{\infty k}} - \beta_k\right) n_k + \sum_l \lambda_l m_{lk}$$

and

$$\dot{m}_{lk} = \beta_{lk} n_k - \lambda_l m_{lk} \quad , \tag{34}$$

with the initial conditions

$$n_k(0) = 1.0$$

and

$$m_{lk}(0) = \frac{\beta_{lk}}{\lambda_l} \quad . \tag{35}$$

The coefficients are given below as

$$l_k = \sum_q l_{q,k} \quad ,$$

$$W_k = \sum_q \left(\sum_{q'} WH_{q',q,k} + W_{q,k} \right) = \sum_q \left(\sum_{i \in q,k} \sum_{j \in q'',k} S_j^* W_{ij} S_i^0 \right) \equiv \sum_q W'_{q,k} \quad ,$$

$$W^+_k = \sum_q W^+_{q,k} \quad ,$$

$$W^-_k = \sum_q W^-_{q,k} \quad ,$$

$$WS_k = \sum_q WS_{q,k} \quad ,$$

$$\frac{1}{k_{\infty k}} = \sum_q \frac{1}{k_{\infty q,k}} \quad ,$$

$$\beta_{lk} = \sum_q \beta_{lq,k} \quad ,$$

and

$$\beta_k = \sum_l \beta_{lk} \quad . \tag{36}$$

Note that

$$\sum_{q'} WH_{q',q,k} + W_{q,k} \neq W'_{q,k} \quad ,$$

but summation over q gives the same result. Coefficients other than W_k are obtained from those of the multiregion model.

Equation (34) is the one-dimensional kinetics equation based on a coarse mesh nodal coupling method. The power $P_{q,k}$ [corresponding to Eq. (32)] is given by Eq. (37):

$$P_{q,k} = P_{q,k}(0) n_k \quad . \tag{37}$$

III.C. One-Point Neutronics Model

The one-point neutronics model is thought to be a special case of the axial one-dimensional model. Taking the summation over k in Eq. (34), the following equations are obtained:

$$\dot{n} = \frac{\rho - \beta}{\Lambda} n + \sum_l \lambda_l m_l$$

and

$$\dot{m}_l = \frac{\beta_l}{\Lambda} n - \lambda_l m_l \quad , \tag{38}$$

with the initial conditions

$$n(0) = 1.0$$

and

$$m_l(0) = \frac{\beta_l}{\lambda_l \Lambda} \quad . \tag{39}$$

Here, the coefficients are defined as

$$F = \sum_{q,k} (W''_{q,k} + WS_{q,k}) \quad ,$$

$$\rho = \sum_{q,k} \left(W''_{q,k} + WS_{q,k} - \frac{\lambda_0}{k_{\infty q,k}} \right) \Big/ F \quad ,$$

$$\beta_l = \sum_{q,k} \beta_{lq,k} / F \quad ,$$

$$\beta = \sum_l \beta_l \quad ,$$

and

$$\Lambda = 1/F \quad , \tag{40}$$

where $W''_{q,k}$ is related to W_k, W^+_k, and W^-_k by

$$\sum_{q,k} W''_{q,k} = \sum_k (W^-_{k+1} + W^+_{k-1} + W_k) = \sum_{q,k} \left(\sum_{i \in q,k} \sum_j S_j^* W_{ij} S_i^0 \right) \quad . \tag{41}$$

In deriving Eq. (38), the following normalization is imposed on S_i^*:

$$\sum_k l_k = \sum_i S_i^* l_i S_i^0 = 1.0 \quad . \tag{42}$$

The definition of F in Eq. (40) is different from the usual definition in diffusion theory.[12] According to diffusion theory, F should be given as

[12] A. F. HENRY, *Nucl. Sci. Eng.*, **3**, 52 (1958).

$$F = \sum_{q,k} \sum_{i \epsilon q,k} S_i^* S_i^0 \ . \tag{43}$$

Although this definition is more natural in defining β_l and the reactivity ρ, using this introduces an inconsistency between ρ and λ of Eq. (3), namely, ρ is not equal to $(\lambda - 1)/\lambda$. This is because the definition of λ is not consistent with that used in diffusion theory: production/(absorption + leakage). However, this definition does not influence $(\rho - \beta)/\Lambda$ and β_l/Λ.

Equation (38) is the one-point kinetics equation based on a coarse mesh nodal coupling method. The power $P_{q,k}$ [corresponding to Eq. (32)] is given by Eq. (44) as

$$P_{q,k} = P_{q,k}(0)n \ . \tag{44}$$

III.D. Two-Dimensional Cylindrical Coordinates Neutronics Model

The foregoing modeling was with respect to rectangular coordinates. It is possible to reformulate the original three-dimensional kinetic equations [Eq. (13)] in two-dimensional cylindrical coordinates.

An assumption is made that the coefficients of Eq. (13) are constant in the circular node shown in Fig. 2. Equation (13) is integrated over each node. Approximating that the summation over j in the radial direction is proportional to the area of the cylindrical node boundary, the following equations are obtained:

$$\begin{aligned} l_n S'_{n,k} = {} & WR_{n-1,k} S'_{n-1,k} + WL_{n+1,k} S'_{n+1,k} \\ & + W^-_{n,k+1} S'_{n,k+1} + W^+_{n,k-1} S'_{n,k-1} \\ & + \left(2W_{n,k} + WS_{n,k} - \frac{\lambda_0}{k_{\infty n,k}} - \beta_{n,k}\right) S'_{n,k} \\ & + \sum_l \lambda_l C'_{ln,k} \end{aligned}$$

and

$$\dot{C}_{ln,k} = \beta_{ln,k} S'_{n,k} - \lambda_l C'_{ln,k} \ . \tag{45}$$

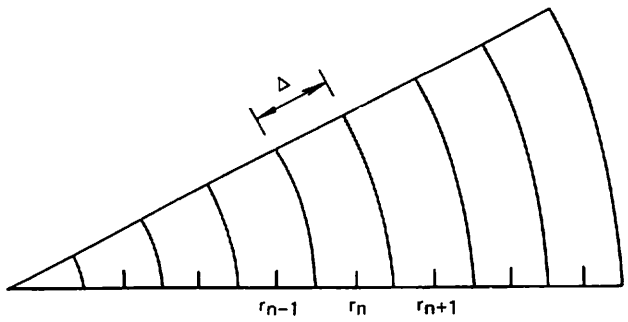

Fig. 2. Node specification for cylindrical coordinates.

Here $S'_{n,k}$ means the number of neutrons that are born in the node n,k (not the source density in unit volume) and the following approximation is further introduced:

$$(r_n \pm 0.5\Delta)/r_{n\pm1} = 1 \mp \frac{\Delta}{2r_{n+1}} \ , \tag{46}$$

where

r_n = radius at the center of node n

Δ = node width.

The coefficients of Eq. (45) are given as

$$WR_{n,k} = W_{n,k;n+1,k}\left(1 + \frac{\Delta}{2r_n}\right) \ ,$$

$$WL_{n,k} = W_{n,k;n-1,k}\left(1 - \frac{\Delta}{2r_n}\right) \ ,$$

and

$$W^-_{n,k} = W^+_{n,k} = W_{n,k;n,k+1} \ . \tag{47}$$

Equation (45) is the two-dimensional cylindrical coordinate kinetics equations based on a coarse mesh nodal coupling method. The power $P_{n,k}$ is given by Eq. (48) as

$$P_{n,k} = \frac{S'_{n,k}\Delta}{2\pi r_n \nu_{n,k}} \ . \tag{48}$$

As a special case, the steady-state, one-dimensional cylindrical equation is given:

$$\begin{aligned} & WR_{n-1} S'_{n-1} + WL_{n+1} S'_{n+1} \\ & \quad + (2W_{n,n+1} + 2W^{\pm}_n \mathrm{NLP} + WS_n) S'_n \\ & = \frac{\lambda}{k_{\infty n}} S'_n \ , \end{aligned} \tag{49}$$

where NLP is the nonleakage probability for axial direction.

IV. METHOD OF SOLUTION

IV.A. Simple Expression of Collapsed Nuclear Constants

The coefficients of kinetics equations derived in Secs. III.A, III.B, and III.C are all defined as integrals over all nodes within region q at axial node k. Since the steady-state, three-dimensional neutron source and absorption importance distributions can be calculated in advance, these coefficients can also be prepared as functions of moderator density u and fuel temperature T for each combination of q and k considering the operating history and control rod configuration.

The following expressions are found to give a good approximation:

$$Y_{q,k} = Y^0_{q,k}[1 + Y^1_{q,k}(u_{q,k} - u^0_{q,k}) + Y^2_{q,k}(u_{q,k} - u^0_{q,k})^2]$$

and

$$k_{\infty q,k} = k^0_{\infty q,k}[1 + A_{q,k}(u_{q,k} - u^0_{q,k}) + B_{q,k}(u_{q,k} - u^0_{q,k})^2] \times \{1 + [C_{q,k} + D_{q,k}(u_{q,k} - u^0_{q,k})] \times [(T_{q,k})^{1/2} - (T^b_{q,k})^{1/2}]\} \quad . \tag{50}$$

Here, all coefficients other than $k_{\infty q,k}$ are denoted as $Y_{q,k}$ for simplicity. The variable $u^0_{q,k}$ is the average value of steady-state moderator density in region q at axial node k, and the variable $T^b_{q,k}$ is the average value in region q at axial node k of the base temperature at which nuclear constants are calculated. Since there are no constraints on the normalization of S^*_i in the previous formulation except for the one-point model, both S^0_i and S^*_i are normalized such that their core average values are 1.0. For the one-point model, the constraint of Eq. (42) can be treated by redefining Λ as $\sum_k l_k/F$.

Note that these coefficients are fitted as functions of $u_{q,k} - u^0_{q,k}$, not as functions of $u_{q,k}$ alone. This is to accommodate possible differences in the steady-state moderator density distribution of the FLARE and any transient thermal-hydraulics model to be coupled with this neutronics model. Any differences in steady-state temperature distribution must be accommodated by another method (see Sec. IV.D).

It has been assumed in deriving the kinetics equations that the control rod pattern remains unchanged after the transient is initiated. However, it is possible to treat approximately the problem in which control rods move during the transients. In this case, these coefficients must be prepared for each of the control rod patterns, assuming the quasi-steady states of the corresponding S^0_i and S^*_i. Coefficients are then selected for use in accordance with the control rod pattern change during the transients.

IV.B. Application of Quasi-Static Approximation and Prompt Jump Approximations

Two approximations can be used in a prompt subcritical slow transient—quasi-static[13,14] and prompt jump. In general, the following discussion is based on Eq. (13).

The neutron source S_i is factorized into an amplitude function $n(t)$ and a shape function $\psi_i(t)$:

$$S_i = n(t)\psi_i(t) \ ,$$

$$n(0) = 1.0 \ ,$$

and

$$\psi_i(0) = S_i(0) \ . \tag{51}$$

Splitting of S_i into two functions implies splitting of Eq. (13) into two coupled equations, one for the amplitude $n(t)$ and the other for the shape $\psi_i(t)$. Equation (51) is substituted into Eq. (13), multiplied by the yet unknown weighting function ω_i, and integrated over all nodes i:

$$\langle \omega_i l_i \psi_i \rangle \dot{n} + \langle \omega_i l_i \dot{\psi}_i \rangle n = \left\langle \omega_i \left[\sum_j W_{ji}\psi_j + \left(WS_i - \frac{\lambda_0}{k_{\infty i}} - \beta_i \right) \psi_i \right] \right\rangle n + \sum_l \lambda_l \langle \omega_i C_{li} \rangle$$

and

$$\langle \omega_i \dot{C}_{li} \rangle = \langle \omega_i \beta_{li} \psi_i \rangle n - \lambda_l \langle \omega_i C_{li} \rangle \ . \tag{52}$$

The following constraint is imposed on ω_i to make the split unique:

$$\langle \omega_i l_i \psi_i \rangle = 1.0 \ . \tag{53}$$

This constraint implies that $n(t)$ contains the main time dependence and $\psi_i(t)$ accounts only for slow space variations. (Note the similarity to the method of Sec. III.C.)

Assuming the quasi-static nature of ω_i and very slow variation of l_i in space and time, the following equations are obtained for the amplitude:

$$\dot{n} = \frac{\rho - \beta}{\Lambda} n + \sum_l \lambda_l C_l$$

and

$$\dot{C}_l = \frac{\beta_l}{\Lambda} n - \lambda_l C_l \ , \tag{54}$$

where

$$F = \left\langle \omega_i \left(\sum_j W_{ji}\psi_j + WS_i\psi_i \right) \right\rangle \ ,$$

$$\rho = \left\langle \omega_i \left[\sum_j W_{ji}\psi_j + \left(WS_i - \frac{\lambda_0}{k_{\infty i}} \right) \psi_i \right] \right\rangle \Big/ F \ ,$$

$$\beta_l = \langle \omega_i \beta_{li} \omega_i \rangle / F \ ,$$

$$\Lambda = 1/F \ ,$$

and

$$C_l = \langle \omega_i C_{li} \rangle \ . \tag{55}$$

[13] K. O. OTT and D. A. MENELEY, *Nucl. Sci. Eng.*, **36**, 402 (1969).

[14] D. A. MENELEY and K. O. OTT, "Fast Reactor Kinetics—QX1 Code," ANL-7769, Argonne National Laboratory (1971).

Equations for the shape are given by

$$l\dot{\psi}_i = \sum_j W_{ji}\psi_j + \left(WS_i - \frac{\lambda_0}{k_{\infty i}} - \beta_i\right)\psi_i - \frac{l_i\dot{n}}{n}\psi_i + \frac{\sum_l \lambda_l C_{li}}{n}$$

and

$$\dot{C}_{li} = \beta_{li}\psi_i n - \lambda_l C_{li} \quad . \tag{56}$$

The quasi-static approximation takes full advantage of the fact that the time variation of the shape function is of lesser importance than the time variation of the amplitude function. Time derivative of ψ_i is therefore neglected:

$$\sum_j W_{ji}\psi_j + \left(WS_i - \frac{\lambda_0}{k_{\infty i}} - \beta_i\right)\psi_i - \frac{l_i\dot{n}}{n}\psi_i + \frac{\sum_l \lambda_l C_{li}}{n} = 0 \quad . \tag{57}$$

The prompt jump approximation is applied to Eq. (54). This approximation neglects the very rapid change of $n(t)$, in the order of Λ s and assumes that $n(t)$ promptly follows the reactivity change.

Under these approximations, the following set of equations is derived for the amplitude and the shape:

$$\dot{C}_l = \frac{\beta_l}{\beta - \rho}\sum_m \lambda_m C_m - \lambda_l C_l$$

and

$$n = \frac{\Lambda}{\beta - \rho}\sum_l \lambda_l C_l \tag{58}$$

and

$$\sum_j W_{ji}\psi_j + \left(WS_i - \frac{\lambda_0}{k_{\infty i}} - \beta_i\right)\psi_i + \frac{\sum_i \lambda_l C_{li}}{n} = 0$$

and

$$\dot{C}_{li} = \beta_{li}\psi_i n - \lambda_l C_{li} \quad . \tag{59}$$

Accordingly, the constraint of Eq. (53) is relaxed and it is only required that the right side be constant. Since this constant can be 1.0, Eq. (53) is also considered valid for the prompt jump approximation.

The optimal weight ω_i can be obtained in two different ways. In both cases, ω_i is optimal in that some integral is stationary with respect to any first-order variation of the shape function. The first case is to derive the conditions for which the reactivity ρ of Eq. (55) becomes stationary with respect to $\delta\psi_i$:

$$\delta\rho = \sum_i \frac{\partial\rho}{\partial\psi_i}\delta\psi_i = 0 \rightarrow \frac{\partial\rho}{\partial\psi_i} = 0 \tag{60}$$

and

$$\frac{\partial\rho}{\partial\psi_i} = \left(\sum_j W_{ij}\omega_j + WS_i\omega_i\right)\frac{1}{\lambda} - \frac{\lambda_0}{k_{\infty i}}\omega_i \quad . \tag{61}$$

In deriving Eq. (61), $\rho = (\lambda - 1)/\lambda$ is used. The stationary condition gives

$$\sum_j W_{ij}\omega_j + \left(WS_i - \frac{\lambda_0\lambda}{k_{\infty i}}\right)\omega_i = 0$$

$$[\rho(0) = 0 \ , \quad \lambda(0) = 1.0] \quad . \tag{62}$$

The second case derives the conditions for which Eq. (52) becomes stationary with respect to $\delta\psi_i$. Using the technique similar to that in Sec. III.A, the following equation is obtained:

$$\sum_j W_{ij}\omega_j + \left(WS_i - \frac{\lambda_0}{k_{\infty i}} - \alpha l_i\right)\omega_i = 0 \ , \tag{63}$$

where α is an asymptotic inverse period.

Both Eqs. (62) and (63) mean that the optimal weight is the quasi-static solution of absorption importance, and it can be approximated by the initial steady-state value S_i^*.

IV.C. Method of Numerical Calculation

The quasi-static approximation makes it possible to solve the amplitude $n(t)$ and the shape ψ_i in different time steps. The amplitude is integrated in every Δt_n time step and the shape in every Δt_m time step, the latter being several times as large as the former. The time mesh specifications are shown in Fig. 3.

Since calculation of ρ and β_l of Eq. (55) requires ψ_i and calculation of ψ_i of Eq. (59) requires $n(t)$ of Eq. (58), which is determined by ρ and β_l, some iterative scheme is required. The numerical calculation proceeds in accordance with the following steps:

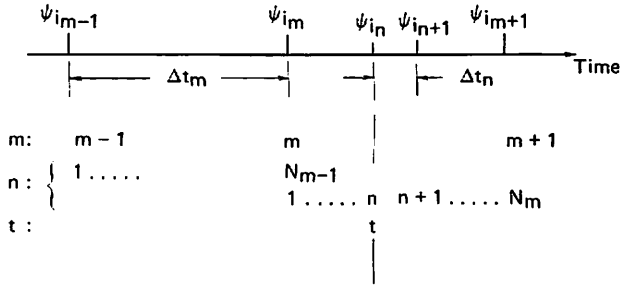

Fig. 3. Time mesh specifications used in the quasi-static approximation.

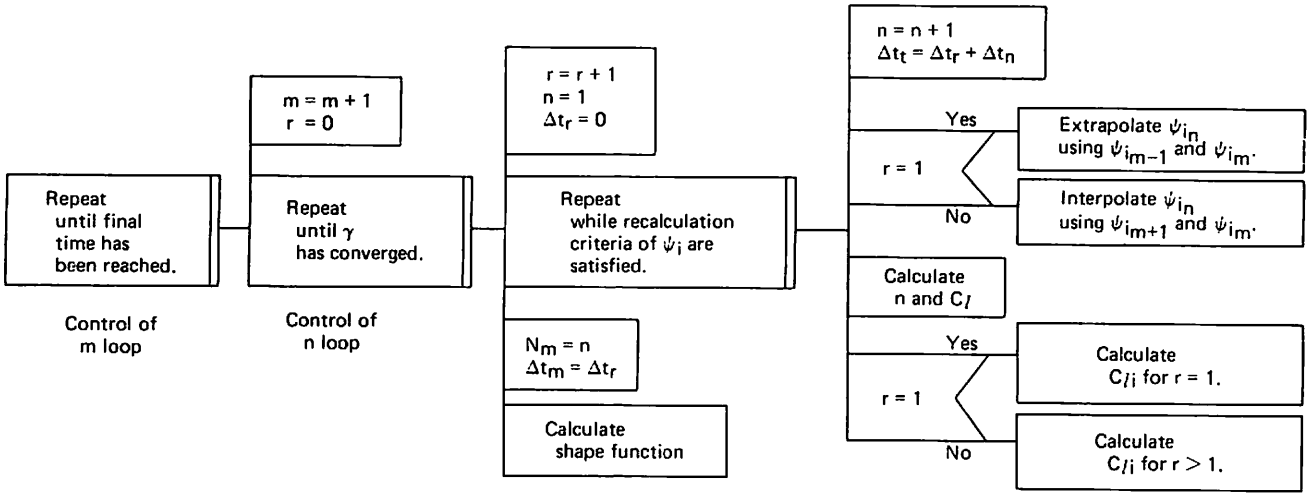

Fig. 4. Iterative scheme for solving the neutronics model by quasi-static approximation.

1. Advance Δt_n.
2. Extrapolate ψ_{i_n} from $\psi_{i_{m-1}}$ and ψ_{i_m} for the first iteration, and interpolate ψ_{i_n} from ψ_{i_m} and $\psi_{i_{m+1}}$ for the succeeding iteration (ψ iteration).
3. Solve n and C_l.
4. Solve C_{li}.
5. Check recalculation criteria of ψ_{i_n} (described later), and either repeat from step 1 or recalculate ψ_{i_n} from n and C_{li}, and set it equal to $\psi_{i_{m+1}}$.
6. Go back to the time when ψ_{i_m} was calculated, and repeat steps 1 through 5 until $\psi_{i_{m+1}}$ is converged.
7. Repeat from step 1.

This iterative scheme is shown by the problem analysis diagram[15] in Fig. 4. The parameter γ is explained later.

IV.C.1. Amplitude Function n

Discretizing the first equation of Eq. (58) at time $t + \frac{1}{2}\Delta t_n$, the following first-order simultaneous equation is obtained:

$$\sum_m^L P_{l,m} C_{m_{n+1}} = \sum_m^L Q_{l,m} C_{m_n} \ , \qquad l = 1, 2, \ldots, L \ , \tag{64}$$

where

$$P_{l,m} = \left(1 + \frac{\Delta t_n}{2}\lambda_l\right)\delta_{l,m} - \frac{\Delta t_n}{2}\frac{\beta_l}{\beta - \rho}\lambda_m$$

and

$$Q_{l,m} = \left(1 - \frac{\Delta t_n}{2}\lambda_l\right)\delta_{l,m} + \frac{\Delta t_n}{2}\frac{\beta_l}{\beta - \rho}\lambda_m \ , \tag{65}$$

where $\delta_{l,m}$ is the Kronecker delta. The reactivity ρ in Eq. (65) is defined as the average at time t_n and t_{n+1} ($= t_n + \Delta t_n$):

$$\rho = \frac{1}{2}(\rho_n + \rho_{n+1}) \ . \tag{66}$$

The delayed neutron precursor fractions β and β_l can be assumed as the values at time t_n.

The amplitude n_{n+1} is obtained by

$$n_{n+1} = \frac{\Delta_{n+1}}{\beta_{n+1} - \rho_{n+1}} \Sigma_m \lambda_m C_{m_{n+1}} \ , \tag{67}$$

where $C_{m_{n+1}}$ is the solution of Eq. (64).

IV.C.2. Delayed Neutron Precursor C_{li}

Integrating the second equation of Eq. (59) over the time interval (t_n, t_{n+1}), the precursor concentration at time t_{n+1} is given as

$$C_{li_{n+1}} = C_{li_n}\exp(-\lambda_l \Delta t_n) + \int_0^{\Delta t_n} \beta_{li} n(t')\psi_i(t') \times \exp[-\lambda_l(t - t')]\, dt' \ . \tag{68}$$

Since β_{li} is assumed to be constant during Δt_n, and ψ_i is assumed to change linearly, Eq. (68) is approximated as

$$C_{li_{n+1}} = D_{l,n} C_{li_n} + \overline{\beta_{li} n_n}(A_{l,n}\psi_{i_n} + B_{l,n}\psi_{i_{n+1}}) \ , \tag{69}$$

[15] Y. FUTAMURA et al., "Development of Computer Programs by PAD (Problem Analysis Diagram)," *Proc. 5th Int. Conf. Software Engineering*, San Diego, California, March 12, 1981, p. 325 (1981).

where

$$\overline{\beta_{li} n_n} = \frac{1}{2}\beta_{li_n}(n_n + n_{n+1}) \ ,$$

$$D_{l,n} = \exp(-\lambda_l \Delta t_n) \ ,$$

$$A_{l,n} = \frac{1}{\lambda_l}[1 - \exp(-\lambda_l \Delta t_n)]\left(1 + \frac{1}{\lambda_l \Delta t_n}\right) - 1 \ ,$$

and

$$B_{l,n} = \frac{1}{\lambda_l}\left\{1 - \frac{1}{\lambda_l \Delta t_n}[1 - \exp(-\lambda_l \Delta t_n)]\right\} \ . \quad (70)$$

Repeating Eq. (69) for $n = 1, 2, \ldots, N_m$, the precursor concentration at time t_{Nm} is given as

$$C_{li_{Nm}} = \prod_{n-1}^{Nm-1} D_{l,n} C_{li_1} + \prod_{n=1}^{Nm-1}\left(\prod_{j=n+1}^{Nm-1} D_{l,j}\right)\overline{\beta_{li} n_n} A_{l,n} \psi_{i_n} + \sum_{n=1}^{Nm-1}\left(\prod_{j=n+1}^{Nm-1} D_{l,j}\right)\overline{\beta_{li} n_n} B_{l,n} \psi_{i_{n+1}} \ . \quad (71)$$

Here, ψ_{i_n} and $\psi_{i_{n+1}}$ must be extrapolated from $\psi_{i_{m-1}}$ and ψ_{i_m} for the first ψ iteration, and interpolated from ψ_{i_m} and $\psi^{r-1}_{i_{m+1}}$ for the r'th ψ iteration. The final expression of $C_{li_{Nm}}$ is, thus, given as

$$C_{li_{Nm}} = \prod_{n=1}^{Nm-1} D_{l,m} C_{li_1} + \sum_{n=1}^{Nm-1}\left(\prod_{j=n+1}^{Nm-1} D_{l,j}\right)\overline{\beta_{li} n_n} \times \left[\left(\frac{\Delta t_r}{\Delta t_m} + 1\right)A_{l,n} + \left(\frac{\Delta t_r + \Delta t_n}{\Delta t_m} + 1\right)B_{l,n}\right]\psi_{i_m} - \sum_{n=1}^{Nm-1}\left(\prod_{j=n+1}^{Nm-1} D_{l,j}\right)\overline{\beta_{li} n_n} \times \left(\frac{\Delta t_r}{\Delta t_m} A_{l,n} + \frac{\Delta t_r + \Delta t_n}{\Delta t_m} B_{l,n}\right)\psi_{i_{m-1}} \quad \text{for } r = 1 \quad (72)$$

and

$$C_{li_{Nm}} = \prod_{n=1}^{Nm-1} D_{l,n} C_{li_1} + \sum_{n=1}^{Nm-1}\left(\prod_{j=n+1}^{Nm-1} D_{l,j}\right)\overline{\beta_{li} n_n} \times \left[\left(1 - \frac{\Delta t_r}{\Delta t_{m+1}}\right)A_{l,n} + \left(1 - \frac{\Delta t_r + \Delta t_n}{\Delta t_{m+1}}\right)B_{l,n}\right]\psi_{i_m} + \sum_{n=1}^{Nm-1}\left(\prod_{j=n+1}^{Nm-1} D_{l,j}\right)\overline{\beta_{li} n_n} \times \left(\frac{\Delta t_r}{\Delta t_{m+1}} A_{l,n} + \frac{\Delta t_r + \Delta t_n}{\Delta t_{m+1}} B_{l,n}\right)\psi^{r-1}_{i_{m+1}} \quad \text{for } r \geq 2 \ , \quad (73)$$

where

$$\Delta t_r = \sum_{i=1}^{n-1} \Delta t_i \ .$$

IV.C.3. Reactivity ρ^r_n

Reactivity ρ_n is calculated by Eq. (55). Since it is time consuming to recalculate Eq. (55) for each ψ iteration, the same approximation as used in Ref. 14 is employed:

$$\rho^r_n = \rho^1_n + (n-1)\frac{\Delta t_n}{\Delta t_{m+1}}(\rho^r_{Nm} - \rho^1_{Nm}) \ , \quad (74)$$

where ρ^r_n means the reactivity at time t_n in the r'th ψ iteration. This approximation can apply also to β_l, β, and Λ.

IV.C.4. Shape Function $\psi^r_{i_{m+1}}$

The shape function at time t_{m+1} for the r'th ψ iteration is obtained by the first equation of Eq. (59) using n^r_{Nm} and $C^r_{li_{Nm}}$ as fixed sources. Note that the constraint of Eq. (53) can be automatically satisfied if $\psi^r_{i_{m+1}}$, n^r_{Nm}, and $C^r_{li_{m+1}}$ are all correctly calculated. Thus, introducing a parameter γ^r (Ref. 14), which is a measure of convergence, $\psi^r_{i_{m+1}}$ is obtained as

$$\psi^r_{i_{m+1}} = \frac{1}{\frac{\lambda_0}{k_{\infty i}}\gamma^r + \beta_i - WS_i}\sum_j W_{ji}\psi^r_{j_{m+1}} + \frac{\sum_l \lambda_l C^r_{li_{m+1}}}{n^r_{Nm}} \ , \quad (75)$$

where

$$\gamma^r = \langle S^*_i l_i \psi^r_{i_{m+1}} \rangle \ . \quad (76)$$

Introduction of γ^r helps stabilize the numerical calculation for a large reactivity insertion in which case the source term becomes very small in Eq. (75).

The shape function should be recalculated when one of the following two relations becomes unsatisfied:

$$\max_i |\psi_{i_n} - \psi_{i_m}| \leq \epsilon_\psi$$

and

$$10^{-(1/\epsilon_n)} \leq \frac{n_n}{n_1} \leq 10^{1/\epsilon_n} \ . \quad (77)$$

Convergence criterion of the ψ iteration is, as stated earlier,

$$|\gamma^r - 1| \leq \epsilon_r \ . \quad (78)$$

IV.D. Special Consideration for Spatial Collapse

IV.D.1. Optimal Weighting Function for Multiregion and Axial One-Dimensional Neutronics Models

The steady-state solutions of Eqs. (29) and (34) are $n_{q,k}(0) = n_k(0) = 1.0$, and they satisfy Eqs. (79) and (80), respectively. Here, the variables are changed from n to S for clarification as

$$S_{q,k} = \frac{k_{\infty q,k}}{\lambda_0}\left[\sum_{q'} WH_{q',q,k}S_{q',k} + W^-_{q,k+1}S_{q,k+1} + W^+_{q,k-1}S_{q,k-1} + (W_{q,k} + WS_{q,k})S_{q,k}\right] \quad (79)$$

and

$$S_k = \frac{k_{\infty k}}{\lambda_0}[W^-_{k+1}S_{k+1} + W^+_{k-1}S_{k-1} + (W_k + WS_k)S_k] \ . \quad (80)$$

The absorption importances corresponding to $S_{q,k}$ and S_k satisfy

$$S^*_{q,k} = \frac{k_{\infty q,k}}{\lambda_0}\left[\sum_{q'} WH_{q,q',k}S^*_{q',k} + W^+_{q,k+1}S^*_{q,k+1} + W^-_{q,k}S^*_{q,k-1} + (W_{q,k} + WS_{q,k})S^*_{q,k}\right] \quad (81)$$

and

$$S^*_k = \frac{k_{\infty k}}{\lambda_0}[W^+_k S^*_{k+1} + W^-_k S^*_{k-1} + (W_k + WS_k)S^*_k] \ . \quad (82)$$

It is evident that the solutions of Eqs. (81) and (82) are $S^*_{q,k} = S^*_k = 1.0$. Therefore, the optimal weighting functions to be used in the quasi-static approximation of Eqs. (29) and (34) are constant. Physically, this means that spatial distribution of importance is already considered in preparing the collapsed nuclear constants.

IV.D.2. Some Means to Account for Introduction of Numerical Errors

Imbalance of reactivity at the initial steady state is inevitable partly due to inconsistency of the fuel heat transfer model (see Sec. IV.A) and partly due to round-off error. Therefore, it cannot be expected that solutions of Eqs. (79) through (82) are perfectly flat.

Eigenvalue λ_0 of the initial steady state is modified by taking the neutron balance in each of the collapsed models, and correction terms $\epsilon_{q,k}$ and ϵ_k are introduced and added to $W_{q,k}$ and W_k to force the solutions to be flat.

1. Multiregion neutronics model:

$$\lambda_0 = \frac{\sum_{q,k}\left(\sum_{q'} WH_{q',q,k} + W^-_{q,k+1} + W^+_{q,k-1} + WS_{q,k} + W_{q,k}\right)}{\sum_{q,k}\frac{1}{k_{\infty q,k}}}$$

$$\epsilon_{q,k} = \frac{\lambda_0}{k_{\infty q,k}} - \left(\sum_{q'} WH_{q',q,k} + W^-_{q,k+1} + W^+_{q,k-1} + WS_{q,k} + W_{q,k}\right) ,$$

and

$$W_{q,k} \leftarrow W_{q,k} + \epsilon_{q,k} \ . \quad (83)$$

2. Axial one-dimensional neutronics model:

$$\lambda_0 = \frac{\sum_k (W^-_{k+1} + W^+_{k-1} + WS_k + W_k)}{\sum_k \frac{1}{k_{\infty k}}} ,$$

$$\epsilon_k = \frac{\lambda_0}{k_{\infty k}} - (W^-_{k+1} + W^+_{k-1} + WS_k + W_k) ,$$

and

$$W_k \leftarrow W_k + \epsilon_k \left(\sum_k \epsilon_k = 0\right) \ . \quad (84)$$

3. One-point neutronics model:

$$\lambda_0 = \frac{\sum_{q,k} (WS_{q,k} + W_{q,k})}{\sum_{q,k}\frac{1}{k_{\infty q,k}}} \ . \quad (85)$$

V. EXAMPLE OF NUMERICAL CALCULATIONS

The models described in previous sections were implemented into computer programs. A collapsed nuclear constant generating program was also prepared separately.

A reference BWR core of 1100 MW(electric) was simulated to verify the effectiveness of the proposed models. Since this paper deals only with the neutronics part, artificial disturbances are imposed on the moderator density and the corresponding neutronics responses are calculated. The initial steady state is a typical low flow-high power condition of BWR (40% flow and 60% power).

Region specifications and a control rod pattern are shown in Fig. 5. The reactor core was radially divided into three regions ($Q = 3$) into two different ways, one by an irregular boundary and the other by a circular boundary, and axially into 24 nodes ($K = 24$).

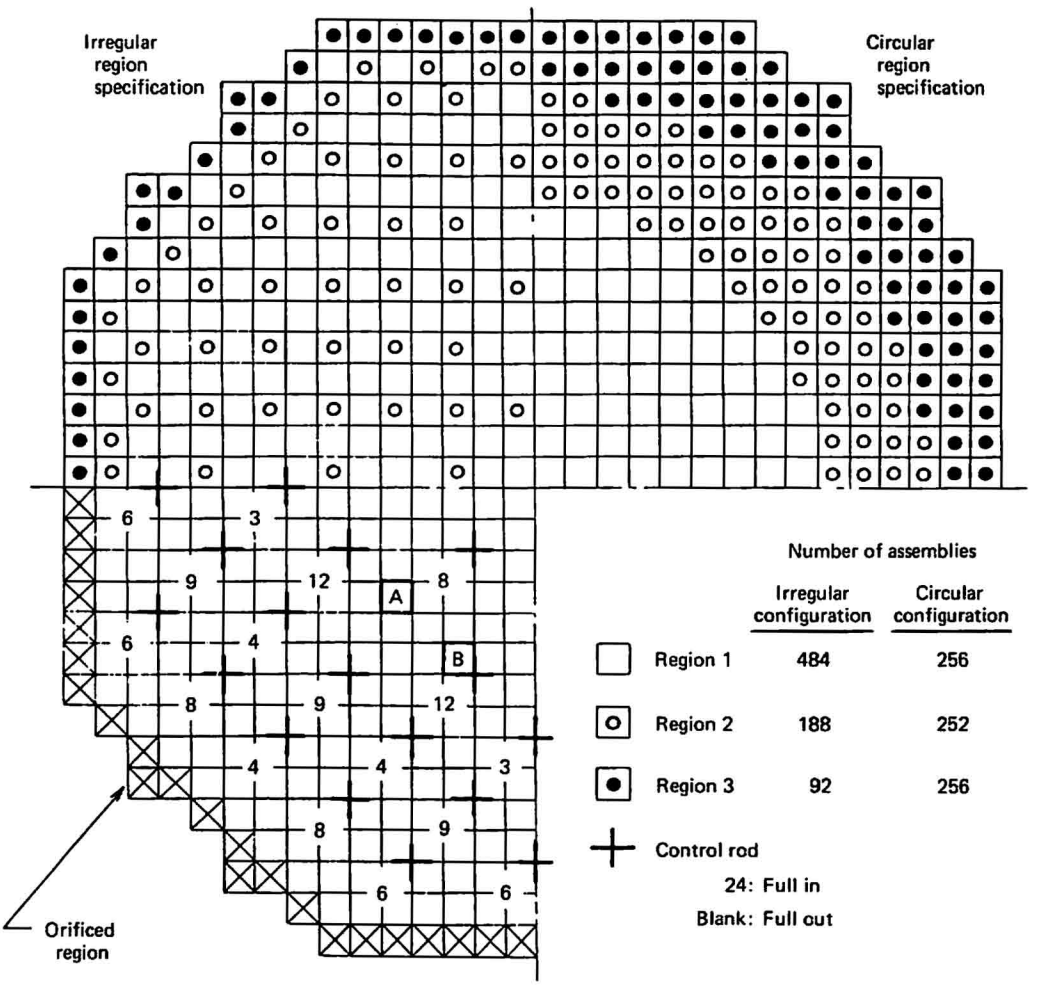

Fig. 5. Region specifications and control rod pattern of reference BWR core.

Steady-state power distributions in these two cases are shown in Fig. 6. The power is peaked in the middle due to the many shallow rods. Two kinds of importance (source importance and absorption importance) are compared in Fig. 7 with source for two selected bundles, A without control rod and B with control rod. The location of these two bundles is given in Fig. 5. It is interesting to note that there is a dip in the source importance around the control rod tip due to its strong absorption characteristics. Since Eq. (1) is not self-adjoint, there is a need to distinguish the importance from the source. It is also important to distinguish the difference of the two kinds of importance.

The collapsed nuclear constants are fitted for the range of $\Delta u = \pm 0.05$ and $\Delta T = \pm 15°C$ around the value of the initial steady state. Here, Δ means the difference from the steady state. A total of 3 X 24 = 72 sets of nuclear constants were prepared, each set containing constants defined in Eq. (31) and following in the form of Eq. (50). The accuracy of the fitting was within $\pm 10^{-3}\%$ and satisfactory.

The three disturbances imposed were all sinusoidal with time, but different with space:

1. uniform throughout core $\Delta u = 0.01 \sin(\pi t)$
2. uniform in radial direction

$$\Delta u = 0.01 \sin\left(\frac{\pi k}{12.5}\right) \sin(\pi t)$$

3. nonuniform in both axial and radial directions

$$\Delta u = 0.01 \sin\left(\frac{\pi k}{12.5}\right) \sin\left[\pi t - \frac{\pi}{2}(q-1)\right] .$$

The principal purpose of these numerical examples is to show the model's capability of treating spatial effect. For this reason, sinusoidal disturbances with a single frequency were selected, although these cannot analyze the cases where the periodicity of the response is not forced by the perturbation and where substantial variation in instantaneous period is possible. The amplitude of the perturbation was set at 1.0% simply because this results in the reactivity of ~20%. These do not indicate the limitation of the model.

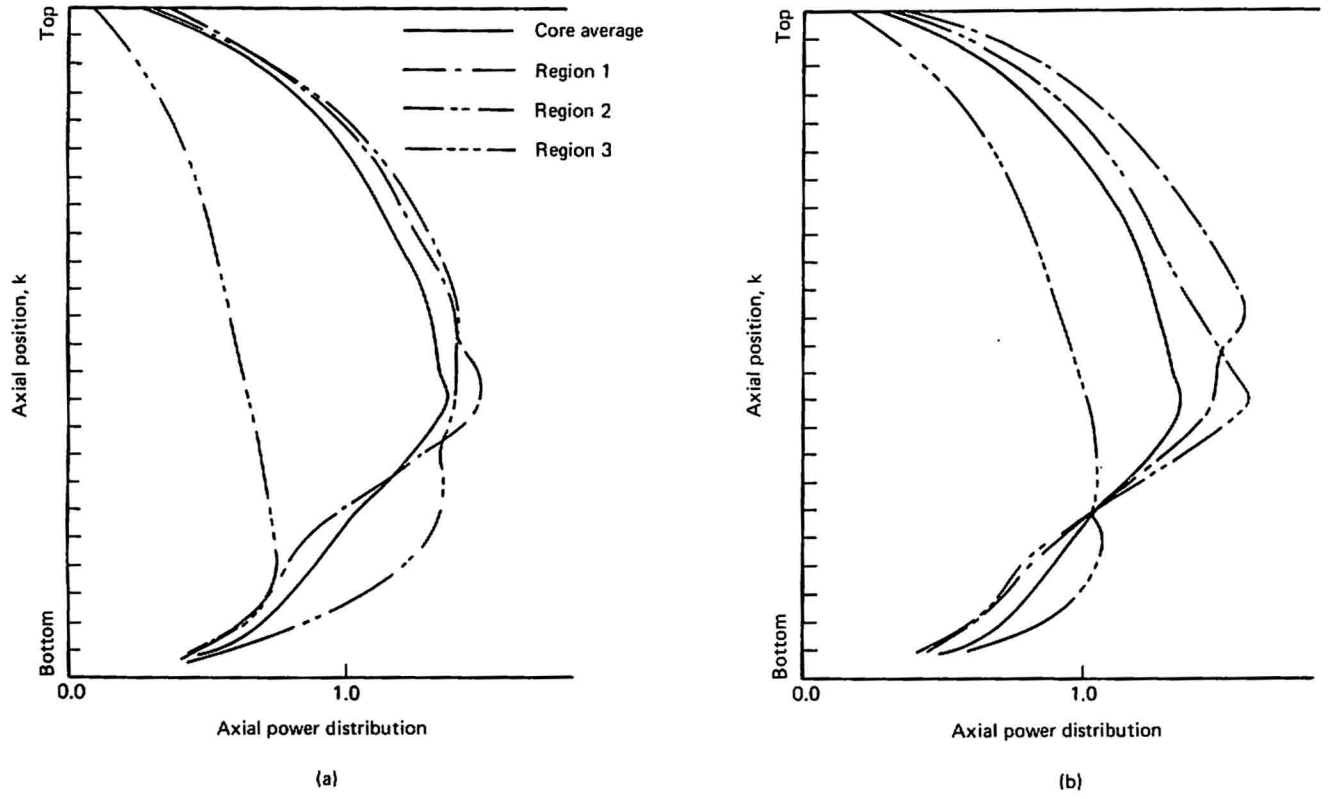

Fig. 6. Axial power distributions at initial steady state: (a) irregular region and (b) circular region.

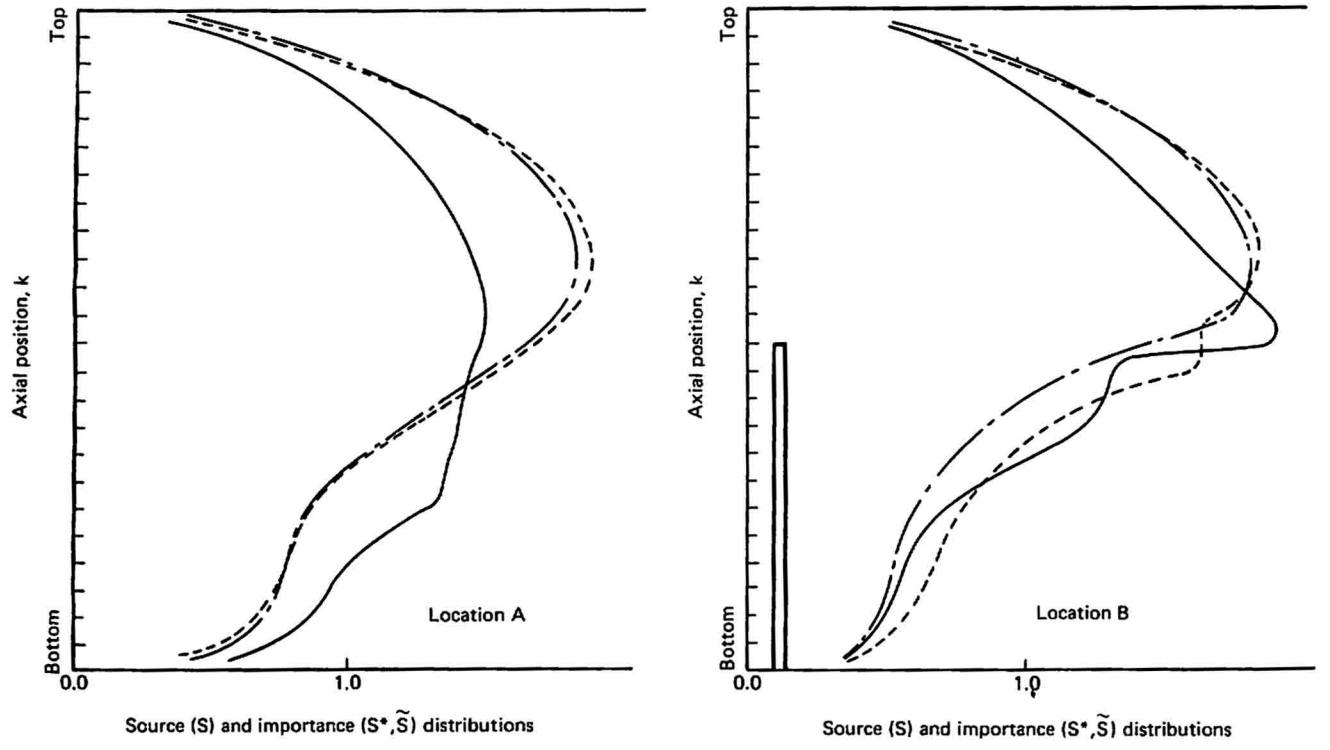

Fig. 7. Comparison of source (S ——), absorption importance (S^* — — —), and source importance ($\tilde{S}$ — — —) distributions for selected bundles (see Fig. 5 for locations A and B).

The transient calculations were performed under the following conditions: Δt_n = 20 ms, ϵ_ψ = 0.005, ϵ_n = 10, ϵ_r = 0.0001, and L = 6. The results are given below for each type of the disturbance.

1. Uniform disturbance throughout core. This is the simplest case. The transient behavior is shown in Fig. 8 for the scatter region. All three models (multiregion, axial one-dimensional, and one point) give essentially the same responses for reactivity and power level. However, the power distribution is periodically distorted along the axial direction due to the void dependency of the infinite multiplication factor ($\partial k/\partial u$ becomes smaller for larger u values). The degree of distortion is ~3%. The results obtained for the circular region are the same. How the core is radially divided does not affect the results for this type of disturbance.

2. Uniform disturbance in radial direction. In this case axial distortion of the power distribution is accentuated more and amounts to as much as 8% as shown in Fig. 9. There is a noticeable difference in both reactivity (1%) and power level (2%) between the one-point model and the other two models. The power distribution behaves the same in the radial direction (see shape function of the multiregion model). These results demonstrate that the axial one-dimensional model is a good approximation for uniform disturbance in the radial direction. The results obtained for the circular region are also the same, as in the previous case.

3. Nonuniform disturbance in both axial and radial directions. This is the case where the net reactivity insertion is small, but the disturbances are nonuniform throughout the core. The results obtained for the irregular region are shown in Figs. 10 and 11, and those for the circular region in Figs. 12 and 13.

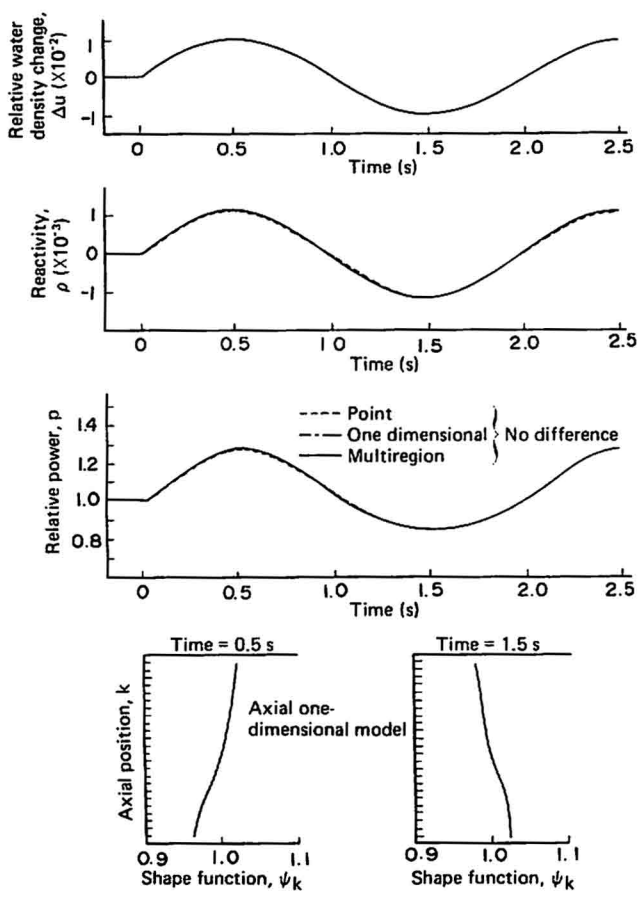

Fig. 8. Transient behavior:
region specification – scatter
disturbance – uniform throughout core
$\Delta u = 0.01 \sin(\pi t)$.

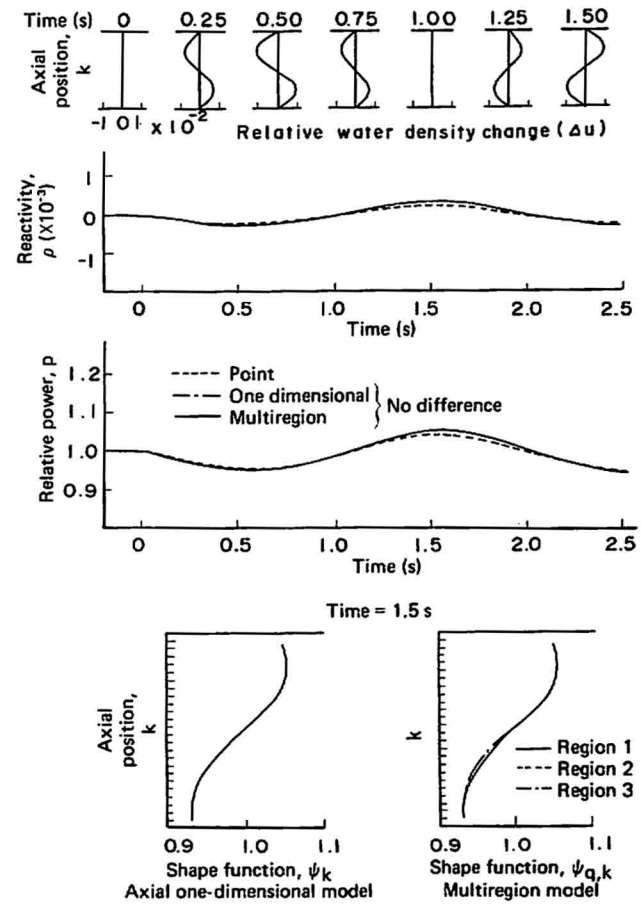

Fig. 9. Transient behavior:
region specification – scatter
disturbance – uniform in radial direction
$$\Delta u = 0.01 \sin\left(\frac{\pi k}{12.5}\right) \sin(\pi t) \ .$$

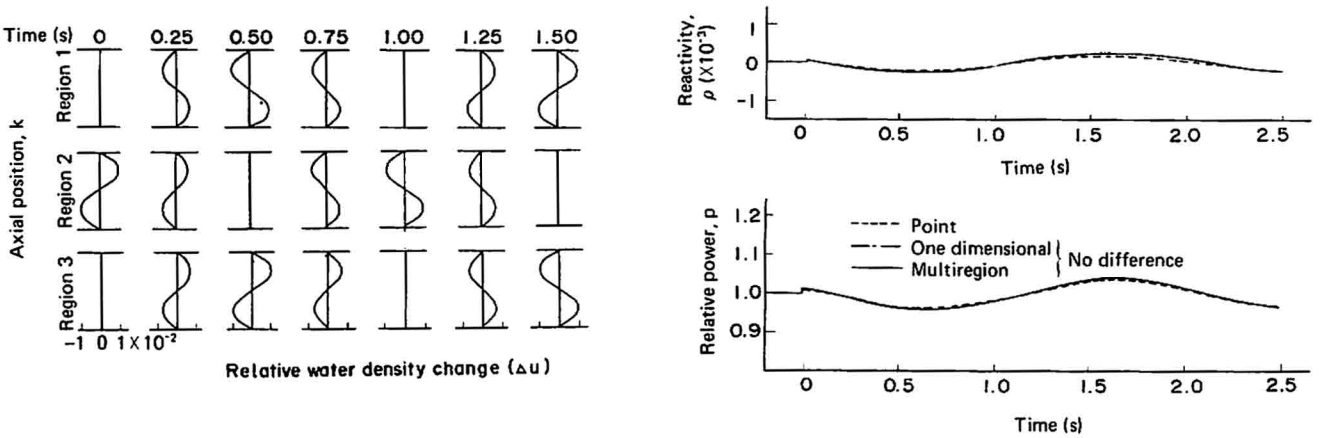

Fig. 10. Transient behavior:

region specification – scatter

disturbance – nonuniform in axial and radial directions

$$\Delta u = 0.01 \sin\left(\frac{\pi k}{12.5}\right) \sin\left[\pi t - \frac{\pi}{2}(q-1)\right] .$$

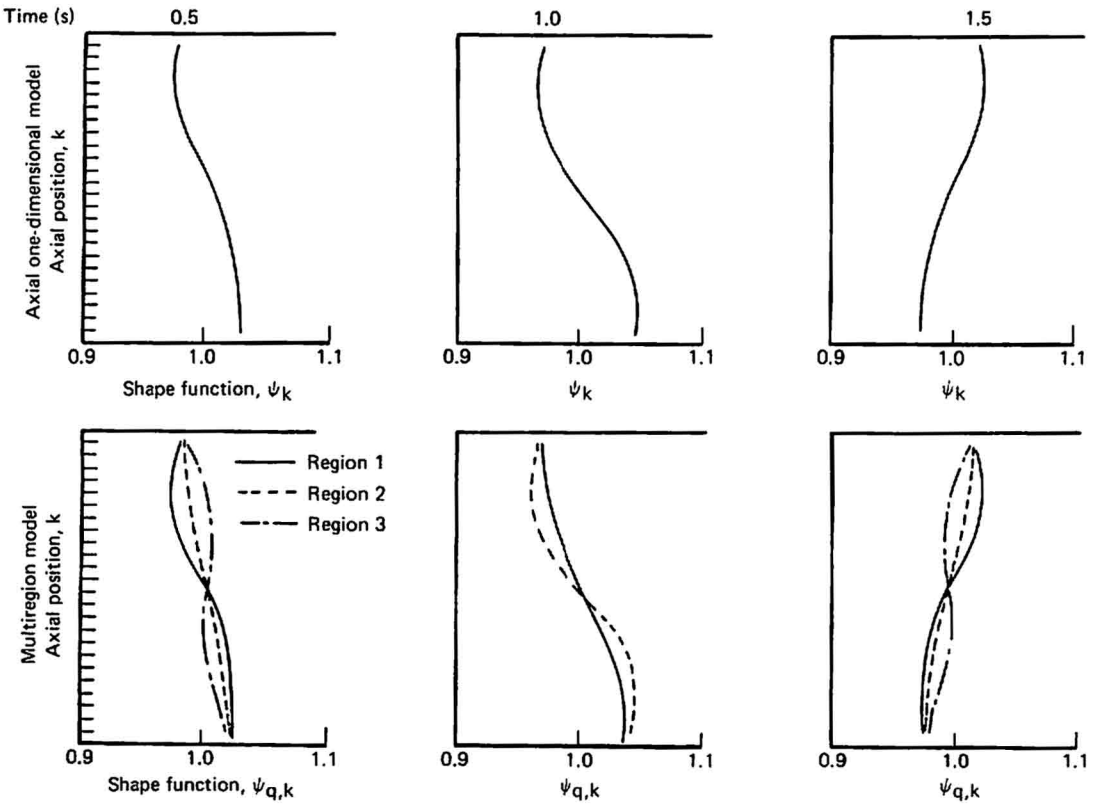

Fig. 11. Change of shape function–axial one-dimensional versus multiregion:

region specification – scatter

disturbance – nonuniform in axial and radial directions

$$\Delta u = 0.01 \sin\left(\frac{\pi k}{12.5}\right) \sin\left[\pi t - \frac{\pi}{2}(q-1)\right] .$$

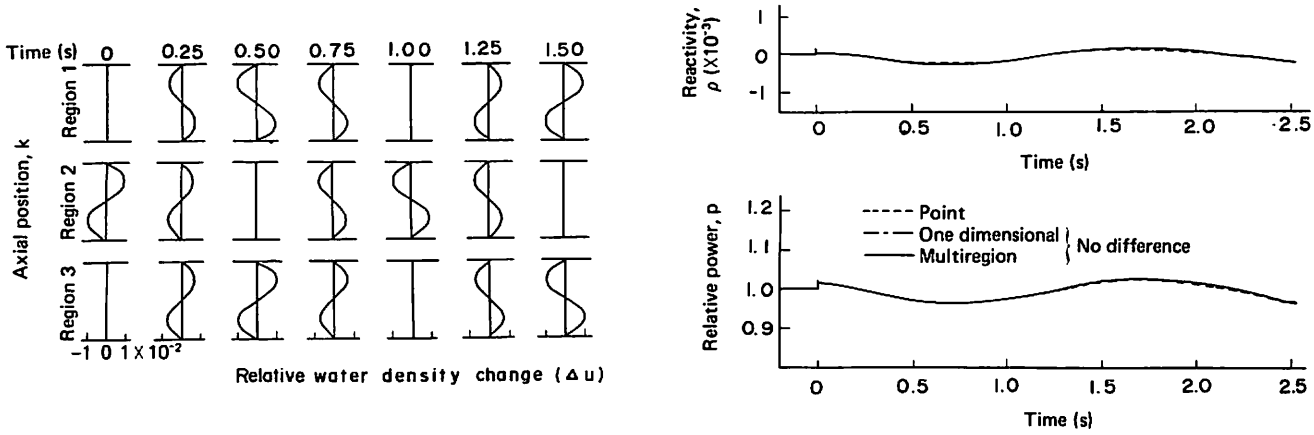

Fig. 12. Transient behavior:

region specification – circular

disturbance – nonuniform in axial and radial directions

$$\Delta u = 0.01 \sin\left(\frac{\pi k}{12.5}\right) \sin\left[\pi t - \frac{\pi}{2}(q-1)\right] .$$

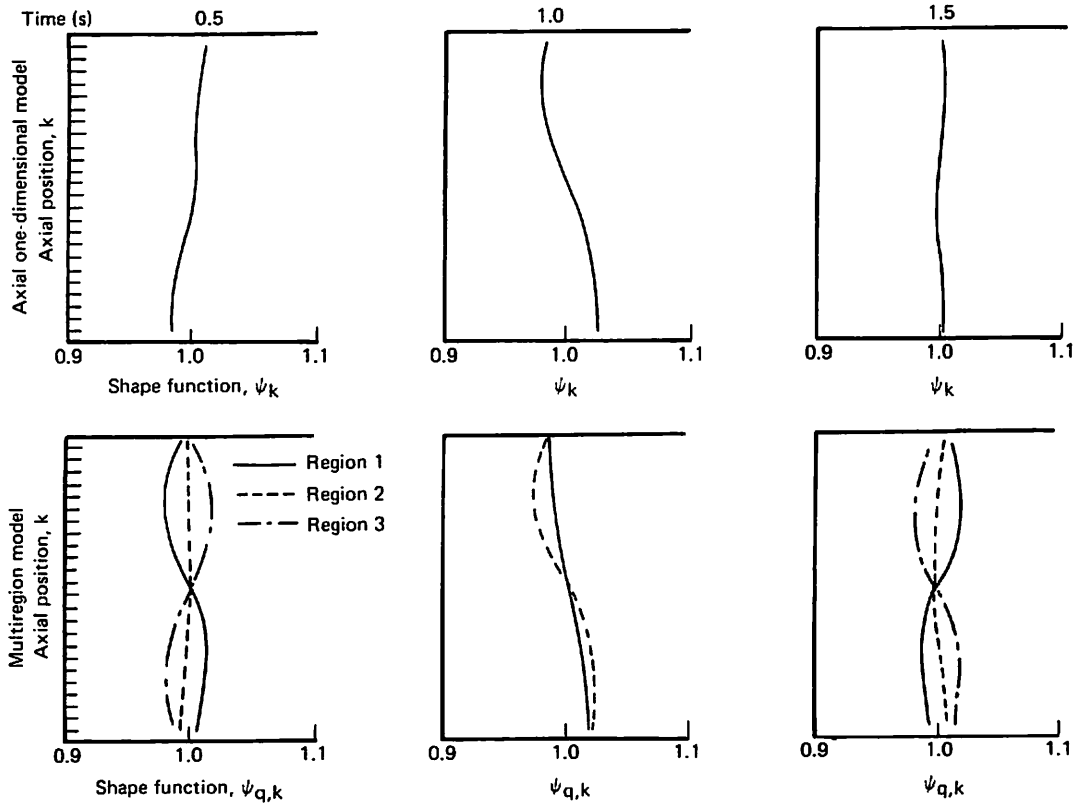

Fig. 13. Change of shape function–axial one-dimensional versus multiregion:

region specification – circular

disturbance – nonuniform in axial and radial directions

$$\Delta u = 0.01 \sin\left(\frac{\pi k}{12.5}\right) \sin\left[\pi t - \frac{\pi}{2}(q-1)\right] .$$

Since the net reactivity change is small in this example, there is not much difference in power level between the one-point model and the other two (no difference is seen between the axial one-dimensional and the multiregion models). However, there is a noticeable difference in power distributions among the three models. The power distributions change out of phase in regions 1, 2, and 3. The regional difference is ~4%.

Note that the scatter region is more closely coupled than the circular region, i.e., the contribution of the

$$\sum_{q'} WH_{q',q,k} n_{q',k}$$

term is much larger for the former in Eq. (29). Therefore, the power distributions in regions 1 and 2 behave more similarily in the scatter region than in the circular region.

These results indicate that the axial one-dimensional model is only good to predict power level. Multiregion treatment is necessary to predict power distributions for this type of nonuniform disturbance.

The results of these examples are reasonable. Computing time required is on the average of 6 s CPU on an IBM 3033 for simulation of a 1-s phenomenon, and it is thought to be practical.

As another example to show the usefulness of this model, a steady-state power distribution of the cylindrical coordinates model [Eq. (49)] is compared to that of the two-dimensional rectangular coordinates model in Fig. 14. The results are in good agreement.

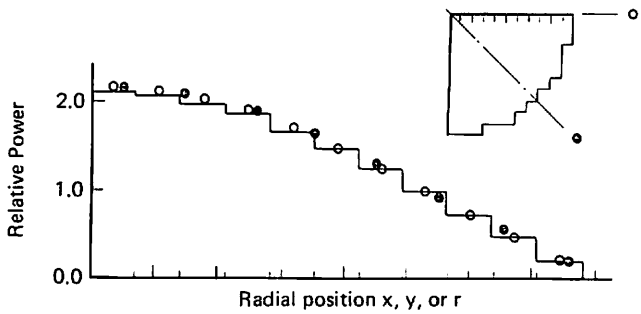

Fig. 14. Comparison of power distributions obtained by cylindrical one-dimensional model (—) and rectangular two-dimensional model (○●).

IV. CONCLUSION

A multiregion neutronics model based on a coarse mesh nodal coupling method is developed for use in transient analysis of a BWR. The method approximates a three-dimensional power distribution by multiregion representation, each region made up of several hundred nodes, and solves its time-dependent characteristics. The axially one-dimensional model, as a special case, and the one-point model, as a further special case of the former, are also formulated.

Two approximations are employed to facilitate numerical calculation—quasi-static and prompt jump. It is proven that the optimal weighting function necessary for the special collapse and for the quasi-static approximation is the steady-state neutron absorption importance and that the use of this function cancels out the effect of the first-order perturbation of the shape function in an integral sense.

These models are implemented into computer programs, and their usefulness is verified by numerical examples applied to a reference BWR core. The results indicate that the axial one-dimensional model is adequate in predicting power level, but inadequate in predicting power distribution for nonuniform disturbances, and the necessity for multiregion neutronics treatment is demonstrated.

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