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Burnup Optimization Using Modal Expansion Method

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A method of modal expansion approximation is applied to study a burnup optimization problem. The spatial distribution of the neutron flux is approximated by a linear combination of certain predetermined spatial modes, and one of these modes is regarded as the control mode. A computational procedure that allows fast and sufficiently accurate estimation of the effect of flux shaping on the attainable burnup is described. As numerical example the optimal policy for flux shaping for a one-dimensional slab reactor model with nonlinear feedback effects is sought by this method. By manipulating the flux shape according to the optimal policy, the attainable burnup is increased appreciably over that obtained by the conventional method based on constant flux distribution, when the maximum allowable power peaking factor is large. The optimal policies are determined uniquely in the cases of highly non-uniform fuel loading and smaller values of the maximum allowable power peaking factor, while they become non-unique in the contrary case. The present method is applicable to more general problems such as the optimization of flux shaping of a reactor with multi-zone refueling scheme without much increase in computing effort.

KEYWORDS*: *burnup, optimization, flux shaping, neutron flux, reactor fueling, nuclear fuels, control rod, computer programming

I. INTRODUCTION

Fuel and poison management of large power reactor cores has considerable influence on power generating cost, and much effort has been directed toward finding the best policy for the management. In past studies, modern optimization theories have been applied for solving the problem with the aid of the digital computer: Wall & Fenech[1] had sought the refueling policy of a three-zone pressurized water reactor core using the technique of dynamic programming. Stover & Sesonske[2] have applied a similar technique to optimize the scatter loading scheme of a three-zone boiling water reactor. Suzuki & Kiyose[3] have investigated refueling optimization on a light-water moderated five-zone power reactor, and have obtained the optimal policy by means of a linear programming technique in combination with the preliminary considerations on a subproblem formulated by stagewise decomposition of the overall problem. In these studies, the problem of poison management optimization was either eluded by assuming that the poison density was varied uniformly in the core, or else only partially treated by assuming that the optimal policy was already known and applied.

Terney & Fenech[4] have made an attempt to solve the problem by making use of dynamic programming and flux synthesis, and obtained a scheme of control rod withdrawal for a radially two-zone reactor. Suzuki & Kiyose[5] have discussed the problem from a general viewpoint using the theory of topological mapping. Motoda & Kawai[6] have presented a geometrical interpretation of the relations among criticality, fuel and poison distribution, power distribution, and fuel burnup in the state space which they named burn-

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** *Ozenji, Kawasaki-shi.*

up space, and determined the optimal control rod motion in this state space. The effect of fuel management was not taken into account explicitly in these studies.

Motoda[7], in his recent work, investigated the coupling effect of fuel and poison management optimization, applying their method of burnup space for a one-dimensional two-zone reactor in equilibrium fuel cycle, and showed that this coupling effect should influence the decision of optimal policy for poison management. It was also shown in his work that the optimization of poison management improved the discharge burnup of the fuel by 0 to 4% over the conventional operation with constant power shape.

All the foregoing studies have been performed utilizing the nodal approximation in which a reactor core was divided into several zones with uniform nuclear properties in each zone. The nodal approach can be considered logical for rendering the problem tractable, especially when the purpose of the analysis is to visualize the effect of the spatial arrangement of materials (*i.e.* fuel and/or poison) within the core. However, when the number of zones to be treated is increased, this approach can no longer be applied to burnup optimization problems, since it then requires too much computing time. This difficulty has so far precluded the possibility of a more general analysis and optimization of the long term operation of a reactor covering both fuel and the poison management.

The aim of the present paper is to develop an alternate approximation procedure that is applicable to optimization problems of a multizone core reactor without suffering restriction by the number of zones. In this method, it is assumed that the influence of control rod programming can effectively be substituted by that of power shaping in relation to the burnup performance of a reactor. The power distribution is expanded into a linear combination of predetermined spatial modes, and one of these modes is regarded as the control mode. The optimal policy for maximizing the discharge burnup of the fuel is determined in terms of the time history of the amplitude of the control mode. The method is applied to a one-dimensional slab reactor with nonlinear feedback effects, and the characteristics of the optimal policy are examined for several typical loading patterns.

II. Statement of the Problem

The behavior of the neutron flux is approximately expressed by the following neutron balance equation for a one-dimensional slab reactor, assuming equivalance of neutron flux and power:

$$\frac{M^2\partial^2\Phi(x,t)}{H^2\partial x^2}+\Big[k(x)-1-\alpha e(x,t)-\beta\Phi(x,t)-\delta\frac{1+\gamma}{\Phi+\gamma}\Phi(x,t)-u(x,t)\Big]\Phi(x,t)=0,\quad 0\leq x\leq 1, \tag{1}$$

where H is the half-width of the core, β and δ are the Doppler and the xenon reactivity feedbacks at the rated power, and γ is a constant such that $(1+\gamma)$ is the ratio of saturated xenon reactivity to the avarage equilibrium value. The symbol u is the reactivity of the control absorber, e the burnup of the fuel, and α the reactivity depletion coefficient for fuel burnup and can be determined arbitrarily. The scale of the normalized time t is decided according to the value of α. The spatial boundary conditions are

$$\left.\frac{\partial\Phi(x,t)}{\partial x}\right|_{x=0}=0$$

$$\left.\frac{1}{\lambda}\frac{\partial\Phi(x,t)}{\partial x}\right|_{x=1}+\Phi(x,t)\Big|_{x=1}=0, \tag{2}$$

where $1/\lambda$ is the linear extrapolation length of the core. In addition, the following relations should be taken into account corresponding to the constant power condition, the constraint on power peaking factor, and the nonnegativity condition of power and absorber:

$$\int_0^1\Phi(x,t)dx=1, \tag{3}$$

$$0\leq\Phi(x,t)\leq\Phi_{\max}, \tag{4}$$

$$0\leq u(x,t) \tag{5}$$

Assuming a linear relation between burnup and flux-time, the burnup distribution is expressed by

$$e(x,t)=\int_0^t\Phi(x,t')dt' \tag{6}$$

or, in differential form,

$$\frac{\partial e(x,t)}{\partial t}=\Phi(x,t),\quad e(x,0)=0. \tag{7}$$

In this study, we consider the problem of maximizing the integral

$$J=\int_0^1 e(x,t_f)dx,\quad t_f:\ \text{undetermined}, \tag{8}$$

which is equivalent to the maximization of the discharge burnup of the fuel, and may be rewritten, simply,

$$\text{minimize } J=-\int_0^{t_f}dt. \tag{9}$$

Equation (7) can be regarded as a system equation and Eqs. (1)~(5) as constraint conditions. Our problem is to find the best control rod programming $u^*(x, t)$, that minimizes the performance index J for the system without violating the constraint conditions. The problem may be treated with the optimization theory for distributed parameter systems. But, direct solution of the problem is, in most cases, quite difficult.

We assume that the distribution of the neutron flux can be approximated by a linear combination of the two known spatial modes:

$$\Phi(x, t)=\omega_1(x)+a(t)\omega_2(x) \tag{10}$$

where $\omega_1(x)$ and $\omega_2(x)$ each represents a fixed component of flux distribution, and must satisfy the spatial boundary condition Eq. (2). In order that the variation of the time dependent coefficient $a(t)$ should not violate Eq. (3), we must impose the condition,

$$\int_0^1\omega_1(x)dx=1, \quad \int_0^1\omega_2(x)dx=0. \tag{11}$$

With above approximations, the time dependent distribution of burnup is simply expressed by

$$e(x, t)=t\omega_1(x)+b(t)\omega_2(x), \tag{12}$$

$$\text{where } b(t)=\int_0^t a(t')dt'. \tag{13}$$

As in Eq. (7), Eq. (13) also is transformed into differential form, leading to the relation

$$\frac{db(t)}{dt}=a(t), \qquad b(0)=0. \tag{14}$$

Hereafter, we regard the shape of the flux distribution, or rather, the time dependent coefficient $a(t)$ as an alternate control variable. The variable $b(t)$ then becomes the state variable, since this variable, and this only, represents the accumulated effect of the control action $a(t)$ on the physical state of the reactor. The temporal change of the state of the reactor is described by Eq. (14) within the validity of the approximation of Eq. (10). The freedom of the system is strongly restricted by the criticality relation, Eq. (1). For the sake of convenience, this equation is solved formally for the distribution of control poison $u(x, t)$, and is rewritten

$$\begin{aligned}u(x, t)=k(x)-1-\alpha[\omega_1(x)t+b(t)\omega_2(x)]\\ -\beta[\omega_1(x)+a(t)\omega_2(x)]\\ -\delta\frac{(1+\gamma)[\omega_1(x)+a(t)\omega_2(x)]}{[\gamma+\omega_1(x)+a(t)\omega_2(x)]}\\ \cdot\frac{M^2}{H^2}\frac{\partial^2[\omega_1(x)+a(t)\omega_2(x)]/\partial x^2}{\omega_1(x)+a(t)\omega_2(x)}.\end{aligned} \tag{15}$$

Obviously $u(x, t)$, the distribution of poison reactivity, is determined uniquely by specifying the value of the three variables a, b and t. The distribution must satisfy the non-negativity condition, Eq. (5), resulting in a restriction on the variables (a, b, t) in a manner expressed in general form by

$$G_1(a, b, t)\geq 0, \tag{16}$$

which indicates the fact that the freedom in flux shaping is influenced by the burnup accumulation in the reactor core. Another constraint, Eq. (4), is transformed into the expression

$$G_2(a)\geq 0, \tag{17}$$

and is rewritten into the simpler form

$$a_{\min}\leq a\leq a_{\max}. \tag{17'}$$

The set of admissible controls is defined by the intersection of the regions satisfying the above two inequalities:

$$G_3(a, b, t)\geq 0, \tag{18}$$

which is reducible to the form

$$a_L(b, t)\leq a\leq a_G(b, t). \tag{18'}$$

If the constraint is satisfied for the specified value of (a, b, t), we can assure ourselves that the reactor is kept critical for that state of the burnup and flux distribution. In other words, given the flux and the burnup distribution in terms of (a, b, t), the distribution of control reactivity can be determined exactly by Eq. (15), and the criticality can be maintained only when the resulting poison distribution is physically realizable.

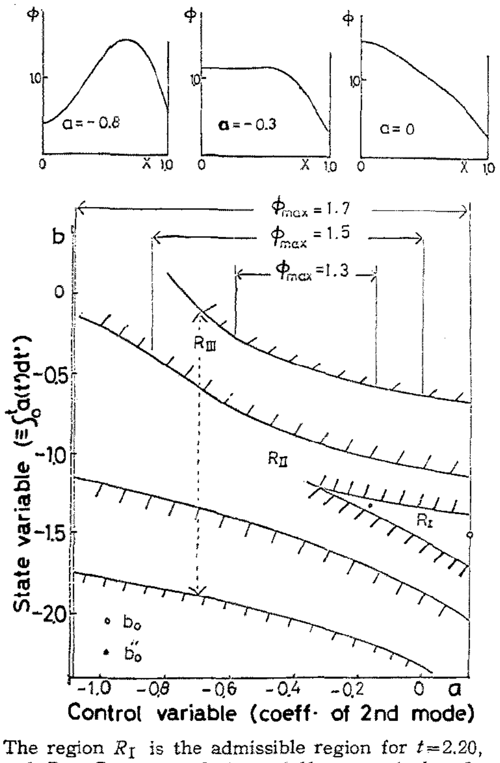
After these approximations and assumptions, our original problem is transformed into compact form, and is described as follows.

Find the optimal control $a^*(t)$ that minimizes the functional J of Eq. (9), subject to the system Eq. (14) and the constraint Eq. (18).

III. Procedure for Seeking Optimal Policy

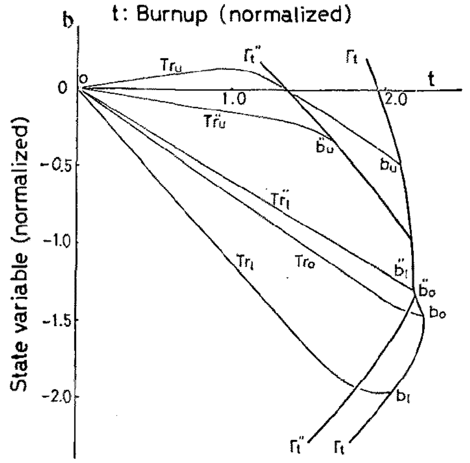
Although our problem has been simplified by the modal approach, it is still difficult to obtain the optimal policy by a purely analytic treatment, or by numerical direct optimization (*i.e.* exhaustive search of all possible control sequences), and some suitable numerical method is called for to seek the optimal policy with less computing effort. A procedure that satisfies this requirement is derived by examining the character of the constraint on the state and the control.

In order to describe the outline of the procedure, a typical pattern of the constraint boundary defined from Eq. (18) is visualized on the (a, b) and (t, b) plane as shown in **Figs. 1 and 2**, using appropriately synthesized spatial modes and parameters equivalent to those given in the next



The region R_I is the admissible region for $t=2.20$, and R_{II}, R_{III} for $t=2.00$, $t=1.60$ respectively. In this figure, and in the others as well, marks with no prime are used in the case of $\Phi_{max}=1.7$, and marks with single and double prime in the case of $\Phi_{max}=1.5$ and 1.3 respectively.

Fig. 1 An example of burnup dependent variation of the admissible region in (a, b) plane.



The boundary trajectories and the target curve define the attainable region $b_l b_u$. The trajectory Tr_0 is one example of many that are possible for arriving at the eligible point b_0.

Fig. 2 Target curves and typical trajectories in $(t-b)$ plane

section. Each contour line in Fig. 1 is obtained by solving

$$G_1(a, b, t)=0 \tag{19}$$

for a fixed value of t. The boundary of the constraint Eq. (17), expressed by

$$G_2(a)=0, \tag{20}$$

becomes two straight lines parallel to the b axis in the (a, b) plane. The relation between the flux shape and a is also illustrated in Fig. 1. The admissible region for every t value defined by Eq. (18) is completely enclosed by these lines and contours, and the range of this region decreases monotonically with increasing t, as might be expected from physical considerations. Within the admissible region, the control variable $a(t)$ can be selected freely, since no other restriction is imposed upon the system.

It is possible to determine the point of the largest t value for every value of the state variable b, so that the non-selfintersecting curve Γ_t is given as shown in Fig. 2.

The following relation holds for all the points on this curve,

$$a_L(b, t)=a_G(b, t) \tag{21}$$

where $a_L(b, t)$ and $a_G(b, t)$ are defined by Eq. (18). Every end of life (EOL) state of a reactor must satisfy one of the limiting conditions of Eqs. (19) and (20), but not necessarily Eq. (21), and therefore some freedom of control is left for the optimization. The reactor can be maintained critical if the residual freedom of control is utilized properly, until this freedom of control diminishes to nothing. On this account, the curve Γ_t is called the target curve, upon which the optimal final state of the reactor must lie. In other words, each point on the target curve can be considered eligible as the optimal EOL state, since, in our optimization problem, there is no reason for stopping the reactor operation before the state reaches the target curve.

It is necessary to distinguish the attainable region on the target curve in order to decide the optimal final state. Two boundary trajectories obtained by the boundary control $a_L(b, t)$ or $a_G(b, t)$ are depicted in Fig. 2, the end points of which lie precisely on the target curve. We call these trajectories the lower boundary trajectory (Tr_l) and upper boundary trajectory (Tr_u). These boundary trajectories and the target curve define a closed region or a sector on the (t, b) plane. Any point within this sector is presumed attainable if the control $a(t)$ is manipulated properly, and hence, the part of the target curve, b_lb_u, is thought to be attainable. In the example of Figs. 1 and 2, the point b_0 is taken for the optimal final point in the case of $\Phi_{\max}=1.7$, since b_0 represents the realizable final state with the largest t value among all terminal points, and the same for the point b_0'' in the case of $\Phi_{\max}=1.3$.

The point b_0 appears attainable by several trajectories, since it lies within the limits bounded by Tr_l and Tr_u with a fair margin, so that the corresponding control policy is not determined uniquely. On the other hand, the point b_0'' can not be reached because it is located outside the limit marked by Tr_l'', and the point b_l'' must be accepted instead as the the most favorable among attainable final states. The optimal control is determined uniquely in this case, and is given by the boundary control $a_L(b, t)$.

The possibility of the existence of non-unique optimal trajectories makes the problem difficult to treat by ordinary search procedure.

Here, assume that the optimal trajectory is composed of two parts: the earlier part of the trajectory is an operation with constant flux shape (*i.e.* $a(t)=a_s$), and the latter part with boundary control (*i.e.* $a(t)=a_L(b, t)$ or $a(t)=a_G(b, t)$). For brevity of description, the earlier part will be called "inner segment", and the latter "boundary segment". The assumption is clearly justified when the optimal control is unique as in the case of Tr_l'' in Fig. 2. With this assumption, the search for the optimal policy is carried out by the following computational procedure:

Step 1 Define $a_{\max}$ and $a_{\min}$ for the specified value of maximum allowable power peaking factor, and determine the increment Δa in such way that $\Delta a=(a_{\max}-a_{\min})/N$, where N is a given integer.

Step 2 Compute the inner segment of the trajectory using the control $a_s=a_{\min}+n\Delta a$ ($n=0, 1, \cdots\cdots, N$) up to the time t_s at which the relation $G_1(a_s, a_st_s, t_s)=0$ holds.

Step 3 Starting from this point (a_s, a_st_s, t_s), compute the upper and the lower boundary trajectory by performing boundary control, until the end point is reached in each trajectory at which the relation $a_L(b_f, t_f)=a_G(b_f, t_f)$ holds.

Step 4 Repeat Steps 2 and 3 for all n, then determine the optimal control $a^*(t)$ to obtain the maximum value of t_f.

By limiting the category of optimal solutions to those stated above, the computational effort is considerably reduced. In addition, the Newton-Raphson algorithm becomes quite effective for the search of the solutions of Eqs. (19) and (21), convergence to the solution being usually obtained with only a few interactions.

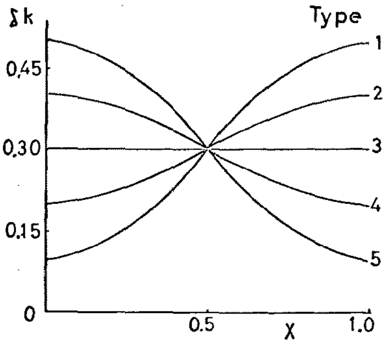
Some quantitative error may be introduced by the simplifications in the procedure. In particular, it would appear inaccurate to compute the two boundary trajectories only, for $t\geq t_s$ in Step 3. However, in general, the freedom in control action is so restricted at this stage of the reactor life compared to that of beginning of life (BOL), that the omission of the non-boundary trajectories should not lead to significant error unless the increment chosen for the control variable is too large. Moreover, it is evident that rough preliminary search of the boundary segment always underestimates the value of the attainable burnup in comparison with the more precise values obtainable with detailed treatment. Thus, the computed results are sure to be more of less pessimistic estimations of the effect of flux shaping, provided sufficient accuracy in the computation of the inner segments. The accuracy of this search procedure will be discussed again in the next section in connection with some numerical examples.

IV. Results and Discussion

A numerical search was performed for a reactor model described in Chap. II, with the parameters of

$$\Delta k=0.3,\ \alpha=0.1,\ \beta=0.007,\ \delta=0.018,$$
$$\gamma=0.3,\ M^2/H^2=0.0035 \text{ and } \lambda=10.0,$$

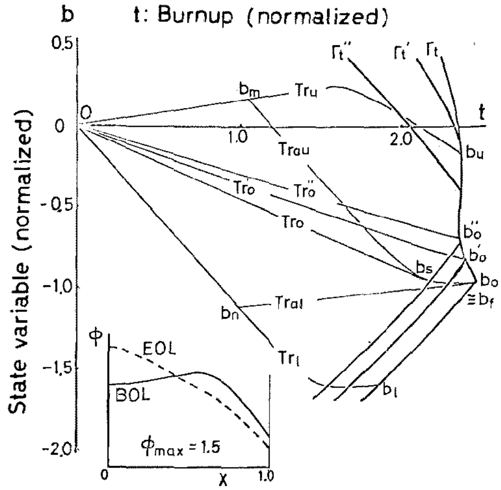
which may be considered representative of a typical boiling water reactor. The effect of the initial loading pattern was studied by taking five initial k-distributions as shown in **Fig. 3** (with notations, Type 1, ……, 5). The maximum allowable power peaking factor $\Phi_{\max}$ was varied from 1.3 to 1.7, for the purpose of examining the influence of this constraint on $\Phi_{\max}$. The spatial modes $\omega_i(x)$ used in this study were derived by means of ordinary Fourier series expansion truncated after the third term, the accuracy of this treatment having already been proved sufficient by one of the present authors[8].



The spatial integration of the loaded fuel $\left(i.e.\ \int_0^1 k(x)dx\right)$ is the same for all loading patterns. The configuration is sinusoidal in all cases.

Fig. 3 Initial loading patterns of fuel considered

The trajectories obtained for a uniformly loaded reactor of Type 3 in Fig. 3 are shown in **Fig. 4**. The target curves for the problem are also shown in order to demonstrate the validity of this reduced search procedure. As the eligible optimal final point, b_0, is found within the attainable region $b_l b_u$ of the target curve Γ_t, it is implied that this point can be attained with some freedom of control, and b_0 is therefore regarded as the real optimal final point. In fact, the best final point



All optimal final points b_0, b_0' and b_0'' are found within the attainable region, and is actually reached by the calculated trajectory Tr_0, Tr_0' and Tr_0'', respectively.

Fig. 4 Optimal trajectories and target curves for fuel-loading Type 3

b_f given by the search procedure comes close to the point b_0 with only a small difference in the attainable burnup (*i.e.* attainable t-value in Fig. 4), demonstrating that the accuracy of the search is sufficient for the present purposes.

By these trajectories, the policy for optimal flux shaping is interpreted, and is transformed into physical terms taking account of the definition of the variables a and b, as follows.

First, the flux is made to keep a shape with a peak in the outer region of the reactor (*i.e.* "outer-high" shape) as shown by the straight line ob_s of the optimal trajectory. The flux shape is held constant up to the time t_s (point b_s) when it becomes impossible to maintain this shape. For $t \geq t_s$, the gradient of the trajectory increases as the time proceeds, implying that the criticality of the reactor is sustained by changing the flux shape as the burnup proceeds, and the peak of the flux is shifted toward the inner region of the reactor (*i.e*, "inner-high" shape). The operation must be terminated at $b_f(\cong b_0)$ where the gradient of the trajectory becomes $a_{\max}$. The flux takes its worst allowable shape at this final point, so that the restriction on $\Phi_{\max}$ must be relaxed for further operation. We call this policy outer-high (O-H) policy. The behavior of the optimal trajectory on the $(t-b)$ plane is the same for all values of

Φ_{max}. This suggests that all optimal policies for flux shaping are qualitatively similar despite the difference in the allowable range for flux variation.

As mentioned already, other optimal trajectories that reach the optimal final point can be synthesized by modifying the computing procedure slightly. Two such trajectories, Tr_{au} and Tr_{al}, are also traced in Fig. 4. These have been obtained by keeping the worst flux shape corresponding to the control a_{max} or a_{min} up to the limiting time t_m or t_n, beyond which the point b_0 would no longer be attainable by any control. These policies however, are not recommendable since they require more sudden change of flux shape than the policy shown by the trajectory Tr_0.

It is notable that the nature of the optimal trajectory is mainly determined by the position of the final point. In the present case for instance, the outer-high flux shape must be maintained for most part of a core life whatever the selected optimal policy may be, since the b-value of the optimal final point is negative with a large absolute value.

An example of the results for non-uniform loading pattern is shown in **Fig. 5**. These trajectories and target curves were calculated for an inner-high fuel distribution, Type 4. The optimal final points lie on a higher part of $(t-b)$ plane compared to that of Type 3, and inner-high flux shape becomes dominant in this situation. The optimal trajectory is defined uniquely and coincides with the upper boundary trajectory Tr_u'' for $\Phi_{max}=1.3$, thus the optimal flux shape is distorted to its utmost toward the inner region.

For $\Phi_{max}=1.5$ and 1.7, the optimal trajectory is not unique, since a slight freedom is admitted to attain the optimal final point, but naturally the overall tendency of the optimal policy is quite similar to the case of $\Phi_{max}=1.3$. We call this policy inner-high (I-H) policy.

The optimal trajectories were calculated and analyzed for all loading patterns in the same way, and the resulting categories of optimal policy are summarized in **Table 1**. It is clearly seen that the optimal flux shaping policy is, as a general rule, the inner-high type for inner-high fuel loadings and the opposite for outer-high fuel loadings. The tendency becomes particularly apparent with increasing non-uniformity of the fuel loading and with decreasing maximum allowable power peaking factor.

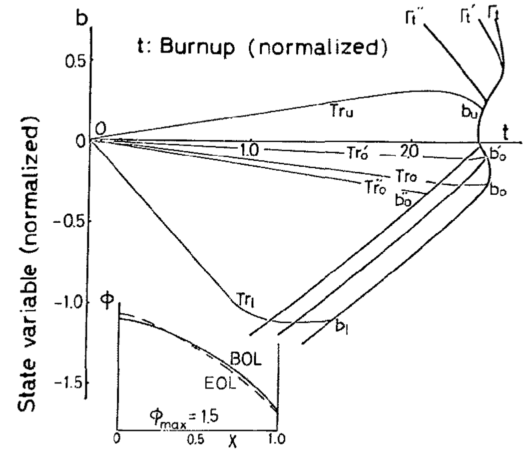


Fig. 5 Optimal trajectories and target curves for fuel-loading Type 4

Table 1 Effect of initial loading pattern and maximum allowable power peaking factor on category of optimal policy for flux shaping

Φ_{max}	Type of fuel loading				
	1	2	3	4	5
1.7	O-H (U)	O-H (N. U)	O-H (N. U)	I-H (N. U)	I-H (U)
1.5	O-H (U)	O-H (U)	O-H (N. U)	I-H (N. U)	I-H (U)
1.3	O-H (U)	O-H (U)	O-H (N. U)	I-H (N. U)	I-H (U)

(U) shows the case of a unique optimal policy, and (N. U) the case of non-unique optimal policies.

The initial value of the flux peaking in the outer (inner) region of the reactor is equal to Φ_{max} for outer (inner)-high policy when the policy is unique, while it becomes smaller than Φ_{max} when the optimal policy is non-unique. For instance, the initial flux peaking is about 1.25 for the optimal policy for the loading pattern Type 3 ($\Phi_{max}=1.7$), while it is equal to Φ_{max} in every case for Types 1 and 5.

The quantitative effect of flux shape optimization is estimated by comparing the values of the attainable discharge burnup with those given by the conventional policy for flux shaping (*i.e.* operation with constant flux shape), and the relative values of burnup improvement obtained are listed in **Table 2**. The amount of improvement is seen to be appreciable for larger values of Φ_{max}, while it is negligible for $\Phi_{max}=1.3$.

Table 2 Improvement of discharge burnup by optimal control
(relative to the maximum burnup obtainable by constant flux shape operation)

Φ_{max}	Type of fuel loading				
	1	2	3	4	5
1.7	5.3%	3.3	2.7	2.8	0
1.5	1.2	2.5	1.2	1.3	0
1.3	0	1.1	1.0	0	0

The conventional policy is thus a good approximation for the optimal policy when the constraint on the maximum allowable power peaking factor is strict, but may bring significant losses when this constraint is not restrictive, and optimization of flux shaping may in such cases provide appreciable improvement to attainable burnup.

Concerning optimization of the refueling scheme, disregard of the effect of flux shaping might lead to incorrect results for deciding the best among the examined refueling schemes, though the resulting difference in the measure of performance (*e.g.* power generating cost) would be small in effect.

It can be pointed out that the improvement in discharge burnup is overestimated for some cases. However, as we stated before, this overestimation is not due to any shortcoming of the search procedure itself, but to the simplicity of the mathematical model of reactor employed in this study.

The categories of optimal policy are well consistent with those given by the burnup space method[6] for a one-dimensional two-zone reactor. The fact that similar results have been obtained from both modal and nodal approximations provides a certain amount of proof for the validity of these approaches, since they are regarded as mutually complementary approximation procedures.

V. Conclusion

The influence of flux shaping on the attainable discharge burnup of fuel is studied by a method of modal approximation technique for a one-dimensional reactor. The optimal policy for flux shaping is obtained in terms of optimal variation of the amplitude of one of the spatial modes.

The characteristics of the optimal policies obtained by the present analysis are summarized as follows.

(1) The policy for attaining the maximum burnup becomes unique beyond a certain limit of non-uniformity of the fuel loading and below a certain value of the maximum allowable power peaking factor, and becomes non-unique in the contrary case.

(2) When the optimal policy is unique, the policy is to maintain as long as possible a flux shape that embodies the highest admissible peak in the region of higher k-value. In our terminology, the optimal policy becomes inner (outer)-high policy for inner (outer)-high fuel loading.

(3) When the optimal policy is non-unique, it is usually possible to find a policy whose fundamental characteristics are similar to the unique one (*i.e.* O-H or I-H policy). In this case, however, the initial value of flux peaking is not equal to Φ_{max}, but takes a certain appropriate value smaller than Φ_{max}.

(4) The quantitative effect of the optimization of flux shaping is estimated by comparing the results with those given by the conventional policy. The amount of improvement possible on the discharge burnup becomes appreciable for larger values of Φ_{max}.

These results are in qualitatively good agreement with the results obtained by nodal approximation procedure. The present method is applicable to the problem of deciding the optimal flux shaping in more complicated reactor models (*i.e.* a reactor with multi-zone refueling scheme) without much increase in computing effort. For further improvement of the accuracy of this approach, the use of the flux-synthesis technique in deriving the spatial modes should be promising.

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