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Method for Evaluating the Local Variation of the Neutron Flux

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One method of evaluating the local variation of the neutron flux is described. By regarding the variations in nuclear reactors such as control rod motion, fuel shuffling and refueling as disturbances added to the material buckling, a one-group diffusion equation is solved using the perturbation theory in one-dimensional geometry. This method is applied to an inhomogeneous reactor, which has already been disturbed, where the disturbances are approximated by a number of square pulses small enough for the method to be applicable. Good results are obtained with this method for disturbances equivalent to a reactivity of several percent. Discussion based on several examples are presented on the errors introduced by larger disturbances.

I. INTRODUCTION

In nuclear reactors, the neutron flux distribution changes its form when the material buckling is disturbed by such events as control rod motion, fuel replacement, and fuel loading. In order to assure the safety and economical operation of reactors, it is very important to establish a general correlation between the causes and effects of variations in neutron flux distribution. It is impossible, however, to solve the diffusion equation analytically when the distribution of the material buckling is complicated, which is, unfortunately, often the case. Thus, the development of an approximate method is called for, to determine the correlation by some simple procedure.

The procedure proposed here consists of the following steps. First, the perturbed diffusion equation is solved for a general form of disturbance and is then applied to the special case of a square disturbance. The variation of neutron flux caused by the introduction of a number of square disturbances can then be obtained by the principle of superposition. Based on the result, thus obtained, a simplified method is developed for predicting the local variation of the neutron flux distribution caused by a square disturbance added to a reactor already disturbed by a number of square disturbances.

II. EQUATION TO BE SOLVED

The diffusion equation describes the neutron flux distribution which directly determines the power distribution in a one-group diffusion model. At critical steady state, in one-dimensional geometry, the diffusion equation of a homogeneous reactor takes the form :

$$\frac{d^2\varphi_0}{dx^2}=\lambda_0\varphi_0(x),\quad \lambda_0=-B_0{}^2,\ \varphi_0(\pm a)=0. \tag{1}$$

Here, $\varphi_0(x)$ means the neutron flux, λ_0 the eigenvalue, and B_0 is the material buckling defined by

$$B_0{}^2=\frac{\delta k_0}{M^2}=\frac{k_\infty-1}{M^2}, \tag{2}$$

where k_∞ is the infinite multiplication factor of neutrons and M^2 the migration area. When the material buckling is disturbed, the equation changes in form to

$$\frac{d^2\varphi(x)}{dx^2}+\delta B^2(x)\varphi(x)=\lambda\varphi(x),\quad \varphi(\pm a)=0, \tag{3}$$

where $\delta B^2(x)$ is the additional buckling, which usually takes the form of square pulse, and λ the eigenvalue after the perturbation. We may write the solution of this equation

$$\varphi=\varphi_0+\delta\varphi \tag{4}$$

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$$\lambda = \lambda_0 + \delta\lambda. \quad (5)$$

The problem is to determine the form of $\delta\varphi$ and $\delta\lambda$.

III. Solution of the Equation

1. Summary of the Perturbation Method

The equation of a state that is not perturbed generally written in the form

$$H_0\varphi_n(x) = \lambda_n\varphi_n(x), \quad (n=0, 1, 2, \cdots) \quad (6)$$

where H_0 is an unperturbed operator, λ_n and φ_n are the eigenvalue and the eigenfunction of H_0, respectively. Here we adopt the assumption that the functions $\varphi_0, \varphi_1, \cdots, \varphi_n, \cdots$ compose a complete set of orthonormal functions. As a rule, the equation of a perturbed state is written

$$(H_0 + \varepsilon V)\varphi(x) = \lambda\varphi(x), \quad (7)$$

where V is the perturbation operator or function of x, and ε a small parameter. The solution of Eq. (7) is expanded into polynomials of ε:

$$\varphi = \varphi_0 + \varepsilon\varphi^{(1)} + \varepsilon^2\varphi^{(2)} + \cdots \quad (8)$$

$$\lambda = \lambda_0 + \varepsilon\lambda^{(1)} + \varepsilon^2\lambda^{(2)} + \cdots \quad (9)$$

Here φ_0 means the solution of the unperturbed state. In order to simplify the ensuring discussion, we adopt the first approximation of the solution, which is given by[1]

$$\varphi^{(1)} = -SV\varphi_0, \quad (10)$$

$$\lambda^{(1)} = (V\varphi_0, \varphi_0). \quad (11)$$

Here, S is an inverse operator of $(H_0 - \lambda_0)$. It should be noted that $(H_0 - \lambda_0)$ has no inverse operator for φ_0, but has it in a subspace which consists of functions orthogonal to φ_0. We define

$$\left.\begin{aligned} S\varphi_m &= \frac{\varphi_m}{\lambda_m - \lambda_0}, \quad (m=1, 2, \cdots) \\ S\varphi_0 &= 0 \end{aligned}\right\}. \quad (12)$$

Thus, the key to the solution is to find a complete set of orthonormal functions and corresponding eigenvalues that satisfy Eq. (6).

2. Superposition of the Solutions

Suppose that the pertubation term of Eq. (7) consists of the numerous functions

$$\left(H_0 + \sum_{n=1}^{\infty}\varepsilon_n V_n\right)\varphi = \lambda\varphi. \quad (13)$$

Then it can easily be shown that the solution of Eq. (13) can be written in a simple form such as given below if the terms of the second order of ε can be neglected.

$$\left.\begin{aligned} \varphi &= \varphi_0 + \sum_{n=1}^{\infty}\varepsilon_n\varphi_n^{(1)} \\ \lambda &= \lambda_0 + \sum_{n=1}^{\infty}\varepsilon_n\lambda_n^{(1)} \end{aligned}\right\} \quad (14)$$

Here, $\varphi_n^{(1)}$ and $\lambda_n^{(1)}$ are the second terms of the right-hand side of Eqs. (8) and (9) respectively, which are obtained when ε_n, V_n are inserted into ε, V in Eq. (7).

3. General Form of the Equation and its Solution

The unperturbed equation

$$\frac{d^2\varphi}{dx^2} = \lambda\varphi, \quad \varphi(\pm a) = 0 \quad (15)$$

has an infinite number of solutions and corresponding eigenvalues. They are classified into two types, one for symmetric and the other for antisymmetric. They are expressed by

$$\left.\begin{aligned} \varphi_{n\cos} &= \sqrt{\frac{1}{a}}\cos\frac{\left(n+\frac{1}{2}\right)\pi}{a}x \\ \lambda_{n\cos} &= -\frac{\left(n+\frac{1}{2}\right)^2\pi^2}{a^2}, \quad (n=0, 1, 2, \cdots) \end{aligned}\right\}, \quad (16)$$

$$\left.\begin{aligned} \varphi_{n\sin} &= \sqrt{\frac{1}{a}}\sin\frac{n\pi}{a}x \\ \lambda_{n\sin} &= -\frac{n^2\pi^2}{a^2}, \quad (n=1, 2, \cdots) \end{aligned}\right\}. \quad (17)$$

In the critical state, the solution of Eq. (15) is φ_0, which is symmetric and belongs to the minimum eigenvalue λ_0. From the procedure given above, it can easily be shown that the perturbed equation cannot be solved unless the product $V\varphi_0$ has the form represented by Eqs. (16) and (17). Then the perturbation V should have the forms

$$\cos\frac{k\pi}{a}x, \quad \sin\frac{\left(k-\frac{1}{2}\right)\pi}{a}x, \quad (k=1, 2, \cdots).$$

If V is given in another form, we must begin by expanding it into these series of functions. Next we solve the equation

$$\frac{d^2\varphi}{dx^2} + \left(\varepsilon\cos\frac{k\pi}{a}x\right)\varphi = \lambda\varphi, \quad \varphi(\pm a) = 0. \quad (18)$$

The solutions of Eq. (18) are calculated from Eqs. (10) and (11):

$$\left.\begin{aligned}\varphi^{(1)}&=\frac{a^2}{4\pi^2}\sqrt{\frac{1}{a}}\cos\frac{3\pi}{2a}x\\ \lambda^{(1)}&=0,\quad (k=1)\end{aligned}\right\}, \tag{19}$$

$$\left.\begin{aligned}\varphi^{(1)}&=\frac{1}{2}\sqrt{\frac{1}{a}}\left[\frac{a^2}{k(k+1)\pi^2}\cos\frac{\left(k+\frac{1}{2}\right)\pi}{a}x\right.\\ &\qquad\left.+\frac{a^2}{(k-1)k\pi^2}\cos\frac{\left(k-\frac{1}{2}\right)\pi}{a}x\right]\\ \lambda^{(1)}&=\frac{1}{2},\quad (k\neq 1)\end{aligned}\right\}. \tag{20}$$

In the same way, the solutions of the equation

$$\frac{d^2\varphi}{dx^2}+\left(\varepsilon\sin\frac{\left(k+\frac{1}{2}\right)\pi}{a}x\right)\varphi=\lambda\varphi,\quad \varphi(\pm a)=0 \tag{21}$$

are obtained. They are

$$\left.\begin{aligned}\varphi^{(1)}&=\frac{1}{2}\sqrt{\frac{1}{a}}\left[\frac{a^2}{\left\{(k+1)^2-\frac{1}{4}\right\}\pi^2}\sin\frac{(k+1)\pi}{a}x\right.\\ &\qquad\left.+\frac{a^2}{\left(k^2-\frac{1}{4}\right)\pi^2}\sin\frac{k\pi}{a}x\right]\\ \lambda^{(1)}&=0\end{aligned}\right\}. \tag{22}$$

These results are easily extended to more general cases making use of the principle of superposition.

4. Application to Square Disturbance

Since the disturbance added to the material buckling is usually given in the form of square pulse, we will apply these solutions to the case of square pulse perturbation.

Figure 1 shows the shape of a symmetric pair of square pulses, and **Fig. 2** the shape of an antisymmetric pair. The arithmetic mean of these two pairs of pulses gives a single pulse such as shown in **Fig. 3**.

Let a and b represent the half widths of the core and disturbed region, respectively. The pulseheight c is related to the magnitude of the perturbing buckling by

$$c=\frac{\delta k}{M^2}. \tag{23}$$

The symmetric pair in Fig. 1 is expanded in series:

$$V(x)_{\text{sym.}}=\frac{2bc}{a}+\sum_{n=1}^{\infty}\frac{4c}{n\pi}\cos\frac{n\pi d}{a}\cdot\sin\frac{n\pi b}{a}\cos\frac{n\pi x}{a} \tag{24}$$

Similarly, the antisymmetric pair in Fig. 2 is expressed by

$$V(x)_{\text{anti.}}=\sum_{n=1}^{\infty}\frac{4c}{\left(n-\frac{1}{2}\right)\pi}\sin\frac{\left(n-\frac{1}{2}\right)\pi b}{a}\cdot\sin\frac{\left(n-\frac{1}{2}\right)\pi d}{a}\sin\frac{\left(n-\frac{1}{2}\right)\pi x}{a}. \tag{25}$$

The arithmetic mean of these functions represents the single pulse in Fig. 3:

$$\begin{aligned}V(x)&=\frac{bc}{a}+\sum_{n=1}^{\infty}\frac{2c}{n\pi}\cos\frac{n\pi d}{a}\sin\frac{n\pi b}{a}\cos\frac{n\pi x}{a}\\ &\quad+\sum_{n=1}^{\infty}\frac{2c}{\left(n-\frac{1}{2}\right)\pi}\sin\frac{\left(n-\frac{1}{2}\right)\pi b}{a}\\ &\quad\cdot\sin\frac{\left(n-\frac{1}{2}\right)\pi d}{a}\sin\frac{\left(n-\frac{1}{2}\right)\pi}{a}x.\end{aligned} \tag{26}$$

Inserting $V(x)$ into $\delta B^2(x)$ in Eq. (3),

$$\frac{d^2\varphi}{dx^2}+V(x)\varphi=\lambda\varphi,\quad \varphi(\pm a)=0. \tag{27}$$

The solutions of this equation are obtained by

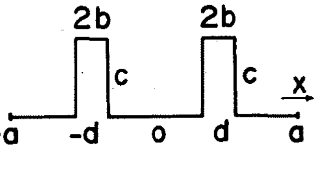

Fig. 1 Symmetric pair of square pulses

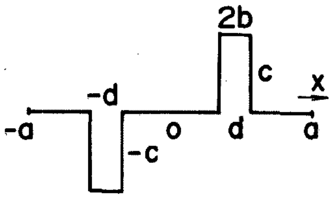

Fig. 2 Antisymmetric pair of square pulses

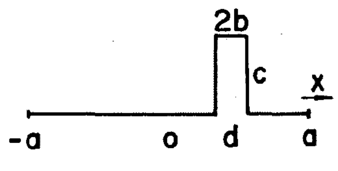

Fig. 3 Single square pulse

using Eqs. (18)~(22) and the principle of superposition:

$$\left.\begin{aligned}\varphi&=\sqrt{\frac{1}{a}}\cos\frac{\pi x}{2a}+\frac{a^2c}{2\pi^2}\sqrt{\frac{1}{a}}\\&\cdot\sum_{n=1}^{\infty}\left(\frac{\alpha_n+\alpha_{n+1}}{n(n+1)}\cos\frac{\left(n+\frac{1}{2}\right)\pi}{a}x\right.\\&\left.+\frac{\beta_{n-1}+\beta_n}{n^2-\frac{1}{4}}\sin\frac{n\pi}{a}x\right)\\\lambda^{(1)}&=\frac{bc}{a}+\frac{c\alpha_1}{2},\qquad \lambda=\lambda_0+\lambda^{(1)}\end{aligned}\right\}\quad(28)$$

Here, α_n and β_n are defined by

$$\left.\begin{aligned}\alpha_n&=\frac{2}{n\pi}\cos\frac{n\pi d}{a}\sin\frac{n\pi b}{a}\\\beta_n&=\frac{2}{\left(n+\frac{1}{2}\right)\pi}\sin\frac{\left(n+\frac{1}{2}\right)\pi b}{a}\sin\frac{\left(n+\frac{1}{2}\right)\pi d}{a}\end{aligned}\right\}\quad(29)$$

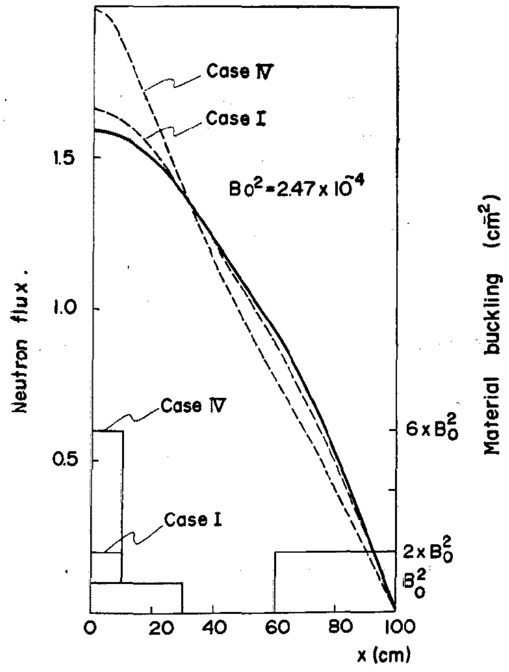

Fig. 4 Comparison between exact and approximated solutions

The combination of this result and the principle of superposition makes it possible to obtain the solution of an equation representing a state of disturbance by a number of square pulses. **Figure 4** shows an example of such calculation, where the summation is carried out for the first ten terms. The results are in good agreement with the exact solution for the small disturbance taken up in this example.

IV. Procedure for Estimating the Local Variation Factor

1. Definition of the Local Variation Factor

The first approximation of the neutron flux disturbed by an additional buckling is represented by

$$\varphi(x)=\varphi_0(x)+\varphi^{(1)}(x). \quad (30)$$

The local variation factor is defined by

$$\beta\equiv\frac{A\varphi(d)}{\varphi_0(d)}. \quad (31)$$

Here, d indicates the position where the perturbation is introduced, and A is a normalizing factor defined by

$$\int_{-a}^{a}A\varphi(x)dx=\int_{-a}^{a}\varphi_0(x)dx. \quad (32)$$

By defining S_0 and ΔS in the forms

$$\left.\begin{aligned}S_0&\equiv\int_{-a}^{a}\varphi_0(x)dx\\\Delta S&\equiv\int_{-a}^{a}\varphi^{(1)}(x)dx\end{aligned}\right\},\quad(33)$$

the constant A is represented by

$$A=\frac{S_0}{S_0+\Delta S}. \quad (34)$$

Accordingly, Eq. (31) becomes

$$\beta=\frac{S_0}{S_0+\Delta S}\left\{1+\frac{\varphi^{(1)}(d)}{\varphi_0(d)}\right\}. \quad (35)$$

2. Local Variation Factor in the General Case

In the preceding section, we have defined the local variation factor in a homogeneous reactor. However, it is more desirable to have an expression of the local variation factor caused by an additional perturbation to a state which has already been disturbed by a number of square disturbances.

In this case, it is expected that the local variation factor depends essentially on the state before perturbation. Then the formulation will be somewhat complicated.

A simple approximate expression of such a local variation factor is obtained by analogy to the homogeneous reactor. But as it might be expected, the method is not useful for the case in which the state before perturbation is remote from the homogeneous reactor.

First we define the neutron flux which has already been disturbed by a number of square disturbances:

$$\psi_n(x)=A_n(\varphi_0(x)+\varphi_1{}^{(1)}(x)+\cdots+\varphi_n{}^{(1)}(x)) \tag{36}$$

Here, A_n is the normalizing factor defined in the same way as before:

$$\left.\begin{aligned} A_n&=\frac{S_0}{S_0+\Delta S_1+\cdots+\Delta S_n}\\ \Delta S_i&\equiv\int_{-a}^{a}\varphi_i{}^{(1)}(x)dx \end{aligned}\right\} \tag{37}$$

When a disturbance is added to this state, the local variation factor is represented by

$$\beta=\frac{\psi_{n+1}(d)}{\psi_n(d)}=\frac{A_{n+1}}{A_n}+\frac{A_{n+1}\varphi_{n+1}^{(1)}(d)}{\psi_n(d)}. \tag{38}$$

In order to simplify the form of β, it is assumed that

$$\psi_{n+1}\approx\psi_n+\varphi_{n+1}^{(1)}. \tag{39}$$

Then β in Eq. (38) becomes

$$\beta'=1+\frac{\varphi_{n+1}^{(1)}(d)}{\psi_n(d)}. \tag{40}$$

The error introduced by this approximation is given by the difference between Eq. (38) and (40), *i. e.*,

$$\beta-\beta'=\left(\frac{A_{n+1}}{A_n}-1\right)+(A_{n+1}-1)\frac{\varphi_{n+1}^{(1)}(d)}{\psi_n(d)}. \tag{41}$$

The first term of Eq. (41) is approximated in the simple form given below, based on the fact that ΔS_i $(i=1, 2, \cdots, n+1)$ are not only negligibly small compared with S_0, but also that they can be either positive or negative:

$$\frac{A_{n+1}}{A_n}-1\approx-\frac{\Delta S_{n+1}}{S_0} \tag{42}$$

The second term of Eq. (41) may be regarded as negligible under the assumption that $\varphi_{n+1}^{(1)}\ll\psi_n$, *i. e.*,

$$(A_{n+1}-1)\frac{\varphi_{n+1}^{(1)}(d)}{\psi_n(d)}\ll 1. \tag{43}$$

Thus, the approximated expression for β is

$$\beta=1+\frac{\varphi_{n+1}^{(1)}(d)}{\psi_n(d)}-\frac{\Delta S_{n+1}}{S_0}.$$

It should be noted that the suffix i $(i=1, \cdots, n)$ is not included except for $\psi_n(d)$ which is given as a known quantity. Then the expression can be simplified to

$$\beta=1+\frac{\varphi^{(1)}(d)}{\psi(d)}-\frac{\Delta S}{S_0}. \tag{44}$$

3. Renormalization of Neutron Flux

The neutron flux is usually given in the form of a peaking factor that is normalized to give an average value of unity. Hence, it is desirable that $\varphi^{(1)}$ and ψ are expressed in the same magnitude. A constant factor γ can be introduced to give ϕ and $\phi^{(1)}$ defined by Eq. (45), such that the average of ϕ becomes unity:

$$\left.\begin{aligned} \phi(x)&=\gamma\times\psi(x)\\ \phi^{(1)}(x)&=\gamma\times\varphi^{(1)}(x) \end{aligned}\right\} \tag{45}$$

Strictly speaking, the second of the two expressions in Eq. (45) is an approximation. From the definition of the peaking factor, the constant γ is defined by

$$\left.\begin{aligned} \phi_0(x)&=\frac{\varphi_0(x)}{\varphi_0(x)_{\text{average}}}=\gamma\varphi_0(x)\\ \gamma&=\{\varphi_0(x)_{\text{average}}\}^{-1}=\frac{2a}{S_0} \end{aligned}\right\} \tag{46}$$

The contribution of the disturbance is, then,

$$\phi^{(1)}(d)=\frac{2a}{S_0}\varphi^{(1)}(d). \tag{47}$$

The local variation factor becomes

$$\beta=1+\frac{\phi^{(1)}(d)}{\phi(d)}-\frac{\Delta S}{S_0}. \tag{48}$$

4. Representation using Parameters

By normalizing the parameters b, d and variable x by

$$\frac{b}{a}=B,\quad \frac{d}{a}=D,\quad \frac{x}{a}=X, \tag{49}$$

the second term of Eq. (28) is transformed into

$$\varphi^{(1)}(X)=\frac{a^2c}{2\pi^2}\sqrt{\frac{1}{a}}\sum_{n=1}^{\infty}\left\{\frac{\alpha_n+\alpha_{n+1}}{n(n+1)}\cos\left(n+\frac{1}{2}\right)\pi X + \frac{\beta_{n-1}+\beta_n}{n^2-\frac{1}{4}}\sin n\pi X\right\}, \quad (50)$$

where α_n and β_n are:

$$\left.\begin{aligned}\alpha_n&=\frac{2}{n\pi}\cos n\pi D\sin n\pi B\\ \beta_n&=\frac{2}{\left(n+\frac{1}{2}\right)\pi}\sin\left(n+\frac{1}{2}\right)\pi B\sin\left(n+\frac{1}{2}\right)\pi D\end{aligned}\right\} \quad (51)$$

Integrating $\varphi_0(x)$ from $x=-a$ to $x=a$, S_0 is obtained:

$$\begin{aligned}S_0&=\int_{-a}^{a}\sqrt{\frac{1}{a}}\cos\frac{\pi x}{2a}dx\\ &=\sqrt{a}\int_{-1}^{1}\cos\frac{\pi X}{2}dX=\frac{4\sqrt{a}}{\pi}\end{aligned} \quad (52)$$

In the same way, ΔS is calculated:

$$\left.\begin{aligned}\Delta S&=\frac{a^2c}{\pi^3}\sqrt{a}\cdot P(B,D)\\ P(B,D)&\equiv\sum_{n=1}^{\infty}(-1)^n\frac{\alpha_n+\alpha_{n+1}}{n(n+1)\left(n+\frac{1}{2}\right)}\end{aligned}\right\} \quad (53)$$

Therefore, the last term of Eq. (48) becomes

$$\frac{\Delta S}{S_0}=\frac{a^2c}{4\pi^2}P(B,D). \quad (54)$$

From Eqs. (45), (46) and (50),

$$\left.\begin{aligned}\phi^{(1)}(d)&=\frac{a^2c}{4\pi}Q(B,D)\\ Q(B,D)&\equiv\sum_{n=1}^{\infty}\left\{\frac{\alpha_n+\alpha_{n+1}}{n(n+1)}\cos\left(n+\frac{1}{2}\right)\pi D + \frac{\beta_{n-1}+\beta_n}{n^2-\frac{1}{4}}\sin n\pi D\right\}\end{aligned}\right\}. \quad (55)$$

Thus we obtain the final form of the local varia-

Table 1 Conﬃcient $P(B, D)$ for local variation factor

B \ D	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
0.05	−0.0562	−0.0507	−0.0356	−0.0145	0.00760	0.0252	0.0340	0.0323	0.0216	0.00780
0.10	−0.110	−0.990	−0.0697	−0.0287	0.0141	0.0482	0.0654	0.0622	0.0423	0.0172
0.15	−0.158	−0.142	−0.101	−0.0425	0.0183	0.0668	0.0915	0.0879	0.0616	—
0.20	−0.198	−0.179	−0.128	−0.0556	0.0195	0.0794	0.110	0.108	0.0795	—
0.25	−0.229	−0.208	−0.149	−0.0680	0.0168	0.0846	0.121	0.121	—	—
0.30	−0.249	−0.227	−0.165	−0.0795	0.00977	0.0817	0.122	0.128	—	—
0.35	−0.258	−0.236	−0.175	−0.0901	−0.00166	0.0706	0.114	—	—	—
0.40	−0.255	−0.235	−0.178	−0.0998	−0.0172	0.0521	0.0990	—	—	—
0.45	−0.242	−0.225	−0.176	−0.109	−0.0363	0.0277	0.0792	—	—	—
0.50	−0.221	−0.207	−0.169	−0.116	−0.0575	0.0000	—	—	—	—

Table 2 Coefficient $Q(B, D)$ for local variation factor

B \ D	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
0.05	0.176	0.188	0.220	0.256	0.279	0.275	0.237	0.169	0.0902	0.0253
0.10	0.305	0.329	0.392	0.466	0.516	0.513	0.442	0.316	0.168	0.0486
0.15	0.393	0.429	0.523	0.633	0.711	0.712	0.618	0.443	0.238	—
0.20	0.443	0.490	0.612	0.758	0.863	0.873	0.761	0.550	0.303	—
0.25	0.461	0.518	0.665	0.843	0.975	0.995	0.875	0.640	—	—
0.30	0.453	0.517	0.686	0.891	1.046	1.080	0.960	0.716	—	—
0.35	0.425	0.495	0.680	0.906	1.082	1.130	1.019	—	—	—
0.40	0.382	0.456	0.651	0.892	1.085	1.149	1.055	—	—	—
0.45	0.330	0.406	0.605	0.854	1.059	1.142	1.073	—	—	—
0.50	0.273	0.348	0.546	0.796	1.009	1.111	—	—	—	—

tion factor:

$$\beta=1+\frac{a^2c}{4\pi}\frac{1}{\phi(d)}Q(B,D)-\frac{a^2c}{4\pi^2}P(B,D). \tag{56}$$

The coefficients $P(B,D)$, $Q(B,D)$ are calculated for some values of B and D. The results are presented in **Tables 1** and **2**.

5. Further Approximation for a Particular Case

It is expected that the variation of the neutron flux will be proportional to the width of the additional square pulse when it is very small compared with the width of the region. This condition, noted by $B\ll 1$, reduces the expression of α, β in Eq. (51) to

$$\left.\begin{aligned}\alpha_n&\approx 2B\cos n\pi D\\ \beta_n&\approx 2B\sin\left(n+\frac{1}{2}\right)\pi D\end{aligned}\right\}. \tag{57}$$

Using this result, the local variation factor is represented by

$$\beta=1+\frac{a^2c}{2\pi}\frac{B}{\phi(d)}R_2(D)-\frac{a^2c}{2\pi^2}B\cdot R_1(D), \tag{58}$$

where, $R_1(D)$ and $R_2(D)$ are defined by

$$\left.\begin{aligned}R_1(D)&\equiv\sum_{n=1}^{\infty}\frac{(-1)^n}{n(n+1)\left(n+\frac{1}{2}\right)}\cdot\{\cos n\pi D+\cos(n+1)\pi D\}\\ R_2(D)&\equiv\sum_{n=1}^{\infty}\frac{\cos\left(n+\frac{1}{2}\right)\pi D}{n(n+1)}\cdot\{\cos n\pi D+\cos(n+1)\pi D\}\\ &\quad+\sum_{n=1}^{\infty}\frac{\sin n\pi D}{n^2-\frac{1}{4}}\cdot\left\{\sin\left(n-\frac{1}{2}\right)\pi D+\sin\left(n+\frac{1}{2}\right)\pi D\right\}\end{aligned}\right\}. \tag{59}$$

These two coefficients, $R_1(D)$ and $R_2(D)$ have been calculated, the result being as shown in **Fig. 5**.

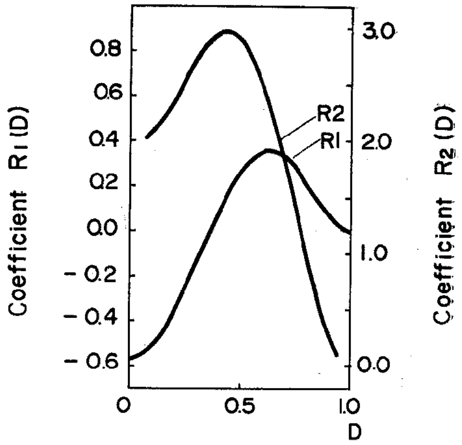

Fig. 5 Coefficients $R(D)$ necessary to estimate local variation factor

V. Results and Discussion

The procedure of estimating the local variation factor is illustrated in **Fig. 6**. It should be noted, however, that the process of deriving Eqs. (56) and (58) includes several approximations. Consequently, the estimation of the errors accumulated thereby is somewhat complicated and is not practical. The validity of these approximations has been otherwise examined through comparison with exact solutions. **Figure 7** shows the exact shapes of the neutron fluxes and the corresponding material buckling distribution. The solid line describes the state adopted as the unperturbed one although it does not correspond exactly to the solution of the homogeneous reactor. The dotted lines represent the shapes of additional bucklings and the corresponding perturbed fluxes. The local variation factors caused by each additional buckling are estimated by using Eq. (56). The results are

Table 3 Exact and approximated local uariation factors

Case No.	Additional buckling	$P(B, D)$	$Q(B, D)$	Non-perturbed flux at $x=0$	Perturbed flux at $x=0$	Exact $L. V.$ factor	Approximated $L. V.$ factor
I	B_0^2	−0.110	0.305	1.585	1.660	1.048	1.045
II	$2B_0^2$	−0.110	0.305	1.585	1.739	1.098	1.089
III	$3B_0^2$	−0.110	0.305	1.585	1.820	1.148	1.134
IV	$5B_0^2$	−0.110	0.305	1.585	1.992	1.257	1.223
V	$10B_0^2$	−0.110	0.305	1.585	2.457	1.550	1.447

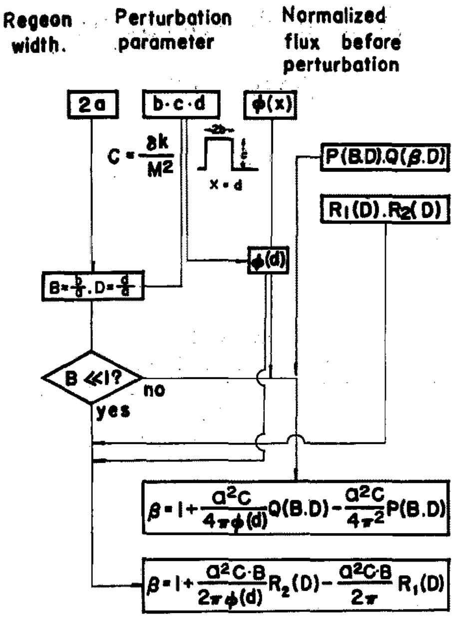

Fig. 6 Illustration of procedure for estimating local variation factor

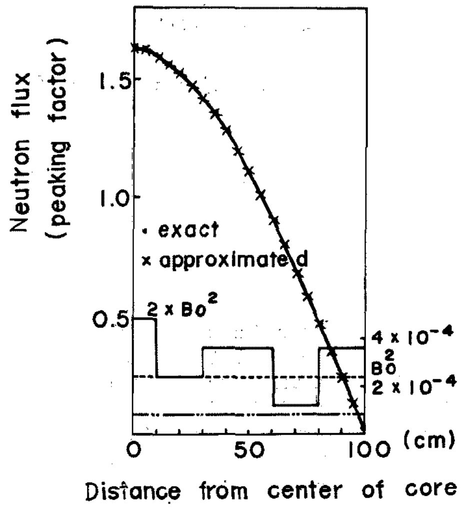

Fig. 7 Change of neutron flux due to disturbances in material buckling

given in **Table 3** together with the exact values. It is clear that the relative error of the quantity $(\beta-1)$ grows with the degree of perturbation. This table indicates the limit to which the present method can be applied and it also reveals the local variation factor to be in good agreement with the solutions obtained from the exact calculations.

——REFERENCE——

(1) TERASAWA, K.: "*Sūgakugairon Ōyō-hen*", (in Japanese), (1961), Iwanami.