frequencies are given in **Table 3**, which also contains those by the consistent scheme.

As seen from these results, the use of lumped bases brings generally some deterioriation in accuracy, but the results are still acceptable and the drawback may well be justified by the considerable saving of the computing process for mass matrices. The lumped basis for plate bending presented here could be utilized in nonlinear dynamic analyses of plates and shells, and its theoretical validity will be reported in due course.

The numerical computations of the present work were carried out by HITAC 5020E computer in the Computer Center, University of Tokyo.

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SHORT NOTE

Investigation of the Fuel Loading Pattern on the Core Burnup by FLARE Simulation

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KEYWORDS*: *optimal loading, fuel loading, burnup, boiling water reactors, zone loading, power peaking, fuel management, poison management, nuclear poisons

Suzuki *et al.*[1] have presented a new method of optimizing fuel shuffling by assuming the stationary property of a region-averaged nuclear constant, *e.g.* k_∞, throughout a reactor life, and have thus succeeded in separating in-core fuel management from poison management. This approach is considered very useful if the assumed nuclear constant is carefully chosen in each region. The optimal k_∞ distribution has been determined by Mélice[2] for a PWR chemical-shim reactor. The present author[3] has solved this problem for BWR by one-dimensional analysis and has shown that the optimal fuel loading pattern consists of three-regions (hereafter called the "optimal principle" in this paper), and that the corresponding control rod programming can thereby be uniquely determined. This optimal loading pattern is very similar to the solution of the minimum critical mass problem[2].

This principle was applied to the initial core of an actual commercial BWR (1,352 MWt, 400 fuel assemblies) and the effect of differences in fuel loading pattern on the core burnup was investigated by FLARE code[4].

The main parameters determining a three-region fuel loading are the following four.

(1) Volume fraction of each region ($v_1, v_2, v_3 = 100 - v_1 - v_2$)
(2) The k_∞ of the fuel in each region ($k_1, k_2, k_3 = (\text{const.} - k_1v_1 - k_2v_2)/v_3$)

Three different combinations of volume fraction (v_1, v_2, v_3) I (33, 33, 34), II (30, 30, 40), III (41, 30, 29) were taken up, which were thought adequate from the previous study[3]. There are six combinations ($_3P_3=6$) of k_∞ distribution. The optimal principle is one which satisfies the relation

$$k_2(=k_{\max}) > k_1 > k_3(=k_{\min}).$$

The k_∞ of the first region (k_1) was determined such that the excess reactivity is canceled at the

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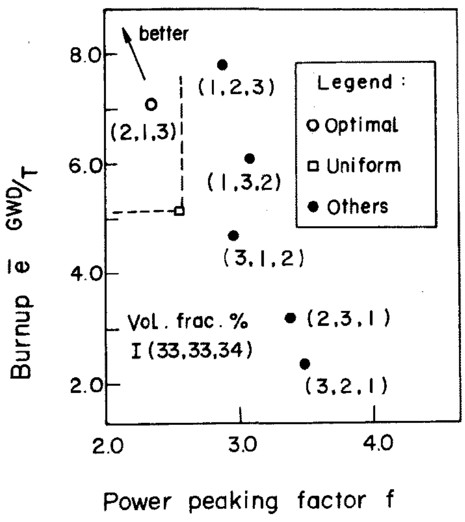
end of life (EOL) by the reactivity decrease caused by fuel burnup. The k_∞ difference, $\Delta k=k_2-k_1$, was varied as a parameter.

It is necessary to determine the control rod programming prior to burnup calculations. It has been shown that the effect of differences in control rod programming is rather small as far as the k_∞ distribution of fuels is near optimum[3] and thus, Haling's principle[5] was adopted. The FLARE code has the capability of calculating this power distribution. It was assumed that the control rods would be withdrawn by certain distances to realize this power distribution at all times throughout the operational period. (Strongly absorbing control rods inserted from the bottom cannot match the actual k_∞ distribution to the ideal target distribution.)

Effect of Optimal Loading Principle

Figure 1 shows the relation between fuel burnup $\bar{e}$ and power peaking factor f for six combinations of three-region loading in the case of $(v_1, v_2, v_3)=(33, 33, 34)$ and $\Delta k=0.08$, and uniform loading. It is evident that the optimal principle (marked ○) has a great advantage for both power peaking and fuel burnup over uniform loading (marked □). The same result holds also for comparisons on other aspects of thermal performance such as minimum critical heat flux ratio.



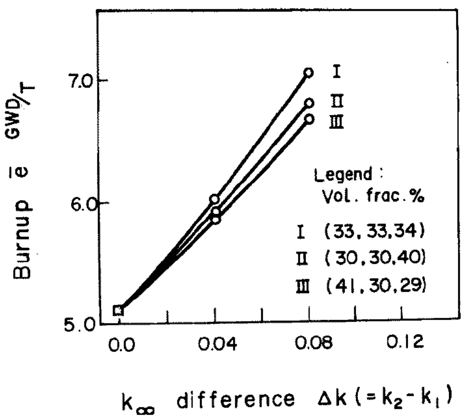
(1 2, 3) *etc.* means the rank of k_∞ among regions. $\Delta k=0.08$.

Fig. 1 Burnup *vs.* power peaking for six combinations of three-region loading

As far as the burnup is concerned, it is best to load the better fuels toward the inner region $(k_1>k_2>k_3)$. However, this results in a very poor power distribution. In other words, the optimal principle indicates loading of the better fuels toward the inner region within the constraint of thermal performance.

Effect of k_∞ Difference between Regions

Figure 2 shows the relation between the k_∞ difference, $\Delta k=k_2-k_1$, and fuel burnup. Since k_2 $(=k_{max})$ is approximately proportional to the core average k_∞ at EOL, the burnup gain $\Delta\bar{e}$ is also proportional to Δk. The quantitative relation between $\Delta\bar{e}$ and Δk is obtained from Fig. 2 as $\Delta\bar{e}$(GWD/T)$=0.25\Delta k$(%). Although the burnup gain is smaller than 1,000 MWD/T for an actual case ($\Delta k\sim$a few %), this amount is still worth consideration. The effect of Δk on the thermal performances, *e.g.* power peak, heat flux *etc.*, is small.



Core averaged k_∞ is kept constant.

Fig. 2 Effect of k_∞ difference between regions

Effect of Volume Fraction

Figure 2 also shows the effect of differences in the volume fraction of each region. The burnup is near maximum when the regions are divided into equal volume.

The radial and the axial power distribution for optimal loading is shown in **Fig. 3**. The radial power distribution in the central region is very flat. Other loading patterns result in poorer power distribution.

In summary, one-dimensional analysis of the optimal loading pattern was verified by a three-

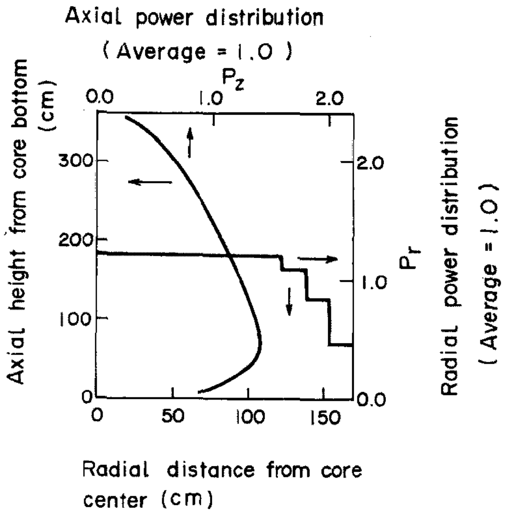


Fig. 3 Power distribution in case of optimal loading (O mark in Fig. 1)

dimensional burnup simulation of BWR. It is concluded that the optimal distribution of the region averaged nuclear constants is very important. This should be carefully determined from the standpoint of both reactor physics and reactor engineering to satisfy the various operational limitations such as stuck rod margin, local power peaking and so on.

Once an optimal k_∞ distribution has been determined, the optimization of the numbers of fuels to be loaded and discharged, and the allocation of the individual fuel assemblies is fairly straight forward(1)(2)(6).

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SHORT NOTE

Electrical Conductivity of Liquid Metal Two-Phase Mixture in Bubbly and Slug Flow Regime

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KEYWORDS*: *liquid metals, electrical conductivity, two-phase flow, mixture, bubbly flow, slug flow, fog flow, MHD generators, NaK-78, void fraction, gamma-ray attenuation, fluid flow

For application to the liquid metal MHD energy conversion cycle, there have been proposed to data four basic cycles, *i.e.* separator, injector condenser, emulsion and slug flow cycles(1).

In the first two cycles, the working fluid flows through the generator channel in liquid phase alone if complete separation or condensation can be obtained of the vapor used to accelerate the

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