Hybrid Simulation of Nuclear Reactor for Optimal Fuel Management

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ABSTRACT

Refueling simulation of the nuclear power reactor was performed using the hybrid computer. This system is composed of the analog computer, the digital computer, the key-board and the color picture tube. The problem of the time-consuming computation of the diffusion equation is solved by using the parallel computation of the analog computer. A successful power computational method was developed and used to decide the refueling schedule by the heuristic method. The study showed that the accuracy and the reproducibility of the diffusion computations by the analog computer was more than enough for the present purpose. The study also showed that the nuclear core was successfully simulated and effectively displayed to find a quasi-optimal refueling order of fuel assemblies of a continuously refueled nuclear reactor.

INTRODUCTION

The continuous refueling which is adopted in Heavy Water Moderated Thermal Reactor has advantages over the batch method in neutron economy and plant availability.

As the fuel burns up, the core reactivity decreases, and the fuel assemblies must be replaced by fresh ones. In a typical reactor of this type, they are replaced one by one in the interval of approximately 7 days among a few hundred assemblies in the core. About one thousand assemblies are refueled during the reactor life.

Among a variety of possible refueling schemes, the optimal method is the one that needs least fuel assemblies during the entire core life. The maximum power density throughout the life must not exceed a prescribed design value to prevent overheating.

The practical difficulty in the optimization of refueling orders lies in the multifariousness of possible methods. For each method, hundreds of two-dimensional diffusion equations must be solved. It is increasingly difficult for a digital computer to handle two and three-dimensional problems, whereas no such difficulty arises in an analog computer as long as the number of mesh points is the same. Therefore, the diffusion calculation is performed by the analog computer in our hybrid simulation.

As the overall optimization of the refueling order is too difficult, the quasi-optimal scheme was searched by the heuristic method. A few possible policies quantitatively expressible at each decision are added with proper weights to give a sum, and the decision is made that maximizes the sum. A series of successive decisions thus made give the resultant average discharge burnup (or the number of fuel assmblies required) at the end of life. Then the burnup is maximized in this weight space.

HYBRID SIMULATION SYSTEM

This hybrid simulation system is composed mainly of three sub-systems. These are the digital computer (HITAC-7250), the analog computer (ALS-1500) and the color display device, respectively, and are functionally organized each other.

This system is represented in Fig. 1. The digital computer is the main part of this system and executes the following items:

1. Set the servo-potentiometers.

2. Take in the digital values obtained through the A-D converter.

3. Normalize the reactor power and calculate the reactivity, the burnup and the fuel depletion.

4. Decide the refueling policy.

The analog computer calculates only the power distribution at each burnup step. The burnup dis-

tribution, the power distribution and some other informations are displayed on the color picture tube.

A necessary information can be put in through the key-board on demand.

REACTOR POWER CALCULATION BY THE ANALOG COMPUTER

Calculation Model

A simplified basic power calculational model is required to be successfully solved by the analog computer. Here, the FLARE model[13] is adopted, which is briefly reviewed below.

Let S_{ijk} be the rate of production of fission energy neutrons at node ijk. If A_{ijk} is the absorption rate at ijk then,

$$S_{ijk} = k_{\infty ijk} A_{ijk} \quad \text{or} \quad S_\ell = k_{\infty \ell} A_\ell, \tag{1}$$

where $k_{\infty \ell}$ is the infinite multiplication factor at the node ℓ. (To simplify writing equations, the single subscript ℓ or m will be used interchangeably with the subscript ijk). Assuming $W_{\ell m}$ as the probability that a neutron born at node ℓ is ultimately absorbed at node m. The absorption rate at node ℓ can then be written

$$A_\ell = \sum_m{}' S_m W_{m\ell} + S_\ell W_{\ell\ell}, \tag{2}$$

where the prime indicates summation over the 2.L nearest neighbors (L is the number of dimension). Combining above two equations leads

$$S_\ell = k_{\infty \ell} \left(\sum_m{}' S_m W_{m\ell} + S_\ell W_{\ell\ell} \right). \tag{3}$$

At the core boundary the neutron leakage from each node is given by

$$\mathcal{L}_\ell = S_\ell W_{\ell m} (n_\ell - \alpha_\ell), \tag{4}$$

where n_ℓ represents the number of external nodes adjacent to node ℓ and α_ℓ is the albedo. The non leakage-probability from any node in general then becomes

$$\begin{aligned} W_{\ell\ell} &= 1 - (2.L - n_\ell)\, W_{\ell m} - (n_\ell - \alpha_\ell)\, W_{\ell m} \\ &= 1 - (2.L - \alpha_\ell)\, W_{\ell m}. \end{aligned} \tag{5}$$

If one divides $k_{\infty \ell}$ by λ (the eigenvalue), the more general form becomes

$$S_\ell = \frac{\frac{k_{\infty \ell}}{\lambda} \left(\sum_m{}' S_m W_{m\ell} \right)}{1 - \frac{k_{\infty \ell}}{\lambda} \left[1 - (2.L - \alpha_\ell) W_{\ell m} \right]} \tag{6}$$

The eigenvalue λ is evaluated, based on a solution to the neutron balance summed over the entire core:

$$\lambda = \frac{\sum_\ell S_\ell - \sum_\ell S_\ell W_{\ell m} (n_\ell - \alpha_\ell)}{\sum_\ell \frac{S_\ell}{k_{\infty \ell}}}. \tag{7}$$

The probability $W_{\ell m}$ can be derived from the one-group diffusion equation for the fast flux φ.

$$M^2 \nabla^2 \varphi + \left(\frac{k_\infty}{\lambda} - 1 \right) \varphi = 0, \tag{8}$$

where M^2 is the migration area. The reactor power P can be approximated, assuming the uniform slowing down property throughout the core, as

$$P = k_\infty \varphi. \tag{9}$$

Comparing the difference equation of Eq. (8), combined with Eq. (9), with Eq. (6), the probability can be approximated as

$$W_{\ell m} \simeq \frac{M_\ell^2}{h_{\ell m}^2}. \tag{10}$$

Calculation Method

There can be thought up a few computational methods based upon the Eq. (6) by using the analog computer. The following two criteria are imposed for the solution of the power distributions.

1. The error of the relative power distributions should be less than 1%.

2. The stable solution can be assured for any sorts of k_∞ distributions.

A few power computational methods have been examined and the following implicit method based on the kinetic version of Eq. (3) is found to satisfy the above criteria. The kinetic equation of Eq. (3) is given as

$$\tau \frac{dS_\ell}{dt} = k_{\infty \ell} \left(\sum_m{}' S_m W_{m\ell} + S_\ell W_{\ell\ell} \right) - S_\ell, \tag{11}$$

where τ is the appropriate time constant.

The significant characteristic of this calculation method is that the higher modes decrease and become negligibly small compared with the foundamental mode in the amplitude after a certain period of time, unless the reactor is critical. The relative power distributions are taken in the digital computer through the A-D converter, at this time, and the eigenvalue is calculated using Eq. (7).

When the reactor is highly super-critical, each nodal power rapidly increases and the output voltage of the operational amplifier may reach its reference value, before the higher modes die away.

This can be accommodated by suitably readjusting the initial values of each nodal power automatically.

Accuracy and Reproducibility

In order to evaluate the accuracy of the solution thus obtained by the analog computer, the same problem is solved by the digital computer using the ordinary method.

The difference of the power distribution is about 3% for the initial core when the core reactivity is large ($\Delta k \simeq 20\%$) and about 0.3% for the equilibrium core when the reactivity is small ($\Delta k \simeq 5\%$). The difference of the eigenvalue is much smaller than the power distribution and within 0.3 % over the entire core life, reflecting the fact that λ is calculated by the overall balance of the neutrons in the core.

As to the reproducibility of the analog solution, the variation of the eigenvalue is within 0.01% and that of the power peaking maximum 3%. This result should be thought satisfactory considering that the 1% variation of the value of the potentiometer for the infinite multiplication factor causes the 10% variation of the power density.

The accuracy and the reproducibility of this much is more than enough for the present purpose.

REFUELING

Heuristic Method

It is very difficult and practically impossible to find the theoretical optimal refueling order among the nearly infinite numbers of possible combinations of refueling patterns by using the recent developed optimization techniques. It is inevitably desirable to simplify the optimization procedure.

The common way of doing this is through the use of so called heuristic rules.[2] A heuristic is any kind of device by which a reasonable, inexact solution may be found to a problem through drastic simplification. A heuristic, to be useful, need only gives a semi-quantitative, and on occasion even only a qualitative approximation to a true or optimal solution. To be useful, it must only avoid bad solutions.

There can be considered certain rules at each stage of decision of refueling order. A linear discriminant function is constructed by summing weighted values of these rules, and these weights are improved or trained by the relation between the rule chosen and the process developed.[3] The rules chosen here are the following two:

1. Discharge the fuel of maximum burnup.

2. Discharge the fuel with minimum power peaking.

The linear discriminant function is thus written as

$$J(\ell) = \frac{e(\ell)}{e_{max}} + w \cdot \frac{P_{f\,max} - P_f(\ell)}{P_{f\,max}} , \qquad (12)$$

where e_{max} and $P_{f\,max}$ mean the maximum allowable discharge burnup and power peaking, $e(\ell)$ and $P_f(\ell)$ are the corresponding values at node ℓ, and w is the weight. At each stage, Eq. (12) is maximized and the ultimate objective function: the average discharge burnup A_f during the entire core life is evaluated,

$$A_f(w) = \frac{\sum_{i=1}^{Num} e^i_{discharge}}{Num} , \qquad (13)$$

where $e_{discharge}$ and Num mean the burnup of the discharge fuel and the number of fuels discharged. The training of the weights w is reduced here to the optimization problem in w space.

Interactive Method

An alternative method is to utilize an intuition of man-kind as a tool for searching the refueling order. The operator can decide the refueling position by his intuition or logical judgement watching the burnup and the power distribution displayed on the color picture tube, as shown in Fig. 1. The interaction is performed through the keyboard.

RESULTS AND DISCUSSION

The above method was applied to the refueling simulation of one-dimensional slab reactor with symmetric 15 regions.

The computing time for the calculation of the power distribution is about 3 seconds. The accuracy and the reproducibility of the power distribution and the eigenvalue are also satisfactory as stated before, and the method is proved to be very effective. However, the problem is found on the automatic setting of the servo-potentiometers. The present mechanism adopts the series settings, rather than the parallel settings, and it takes about 3 seconds for each setting. The number of the servo-potentiometers required is at least twice the number of the regions; one set for adjusting the infinite multiplication factor and another set for adjusting the initial values of the power densities. Therefore, most of the computing time has been consumed in the setting of the potentiometer and after all the computing time for one simulation became about 2 hours.

The refueling trials have been carried out for several sets of $P_{f\max}$ and w through 4 cycles (about 6000 days) both for heuristic and interactive methods. A constant burnup step was used and the refueling decision was made when the core reactivity decreased below the certain limit. No back-burnup was performed for simplicity. One of the results of the reactivity and the power peaking histories is shown in Fig. 2. (B_{max}= 30000 MWD/T, $P_{f\max}$ = 1.75, w = 0.0).

The restriction to the power peaking is found to be very severe for one dimensional problem with not many regions. The heuristic method could not find the solution satisfying the constraints both for burnup and power peaking for small $P_{f\max}$ value. The interactive rules were intuitively determined using the results obtained by the heuristic method, but the result was not so satisfactory. It is felt that the intuition of man-kind is not so powerful for this problem.

Fig. 3 shows the w dependence of the average discharge burnup. The discharge burnup is maximum for w=0.0. This is indeed optimal for larger B_{max}. Fig. 4 shows the P_f dependence of the discharge burnup. The reduction of the constraint of power peaking does not give a significant increase to the discharge burnup.

Some rules can be drawn from the chosen pattern of the refueling order. The order of the refueling often appears cyclicly (3-4-5-2-6) and the corresponding reactivity history also waves with a certain period of cycle time.

CONCLUSION

The power calculation method suitable for the analog computer was developed and applied to the sub-optimization of the continuous refueled reactor by a simplified heuristic method.

The following conclusion can be drawn from the results so far obtained.

1. The power distribution can be calculated successfully by an analog computer for any k_∞ distribution with very short time ($\sim$ 3 sec.).

2. The results obtained as the solution of the heuristic method are reasonable. It is a promising tool for the optimization of a complicated system. Its effectiveness will be shown on the two-dimensional problem with larger numbers of fuel assemblies, using several kinds of rules.

3. It is recommended to include the effect of a few succeeding decisions to the subsidiary discriminant function.

4. The time required for the servo-potentiometer setting must be extremely shortened. This will be achieved by adopting a mechanism of parallel setting or digital potentiometers.

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