

# A Method for Calculating Power Distributions in Boiling Water Reactors Using In-Core Detector Readings

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*A method, using in-core detector readings, is developed for estimating three-dimensional power distributions within boiling water reactors. The power distributions are obtained by a kind of synthesis technique which consists of two types of calculations, i.e., horizontal, two-dimensional whole core, and three-dimensional local core. The detector readings are used as internal boundary conditions.*

*Power distributions obtained by this method compare satisfactorily with the exact solutions from whole-core calculations in test problems. The memory required to implement the technique is also acceptable to a process computer. Hence, the proposed method is suitable for on-line computer application.*

## I. INTRODUCTION

A technique for on-line core performance evaluation in a nuclear power reactor is essential for safe and economical operation. Fixed in-core neutron detectors provide a means for continual monitoring of the core spatial power distributions and the thermal margins to the operational limits.

Since fixed in-core detectors sample only a portion of the active length of the fuel bundle, and most of the fuel bundles are not monitored, an analytical method is required for deducing the whole-core power distribution from a limited number of in-core detector readings. A number of effective methods have been proposed,<sup>1-3</sup> in which the axial power distribution is approximated by a Fourier series expansion of a few modes to match the axial detector readings. The radial power distribution is obtained

either by use of average coupling coefficients that relate the power in unmonitored bundles to the power in the surrounding monitored bundles,<sup>1</sup> or by use of an expansion of a few planar basis functions.<sup>2</sup> Similar methods have been developed by Ryals<sup>4</sup> and Savoia<sup>5</sup> to approximate detailed few-group flux distributions. Melice and Hunin<sup>6</sup> have proposed a scheme whereby the solution of a two-dimensional diffusion equation is corrected to match the measured radial detector signals.

However, these methods are applicable only for pressurized water reactors, in which the approximation of the separability between the axial and the radial power distributions is fairly good. They are not useful for boiling water reactors (BWRs), in which the separability approximation is not valid because of the three-dimensional effects of control rods and steam

<sup>1</sup>R. L. HELLENS, T. G. OBER, and R. D. OBER, *Trans. Am. Nucl. Soc.*, **12**, 820 (1969).

<sup>2</sup>M. M. LEVINE and D. J. DIAMOND, *Nucl. Sci. Eng.*, **47**, 415 (1972).

<sup>3</sup>W. B. TERNEY et al., *Trans. Am. Nucl. Soc.*, **22**, 682 (1975).

<sup>4</sup>H. W. RYALS, *Trans. Am. Nucl. Soc.*, **14**, 750 (1971).

<sup>5</sup>P. J. SAVOIA, *Trans. Am. Nucl. Soc.*, **14**, 751 (1971).

<sup>6</sup>M. MELICE and C. HUNIN, "The CMD Method for PWR In-Core Measurements Processing," *Proc. Specialist Mtg. Spatial Control Problems*, Studsvik, Sweden, October 28-29, 1974, CONF-7410124, International Atomic Energy Agency (1974).

void distribution. For most BWRs equipped with fixed in-core detectors, as well as with axially movable in-core detectors, an extrapolation and fitting method has been applied in which various correlations between the readings and the power values obtained by two-dimensional diffusion calculations<sup>7</sup> are used. The problems with this method are the complicated task of preparing the fitting constants in advance of on-line use for each of reloaded core and the low accuracy for a core operated with asymmetric control rod patterns.

It is possible to estimate the axial power trace using readings of fixed in-core detectors<sup>8</sup> and a simple one-dimensional FLARE-type<sup>9</sup> calculation. The work presented here deals with a method of estimating three-dimensional power distribution in a BWR core using the readings of in-core detectors. It is characterized by a kind of synthesis of horizontal two-dimensional and three-dimensional local core calculations, in which detector readings are used as the internal boundary conditions.

## II. METHOD

The reactor core is divided into a number of cells, each of which consists of four fuel bundles as shown in Fig. 1. About one-fourth of the cells have monitor strings at the center, and there are several neutron detectors equally spaced along the active fuel length of the bundles. These cells are defined as "monitored cells," and the rest, which do not have monitor strings, are defined as "unmonitored cells" in this paper.

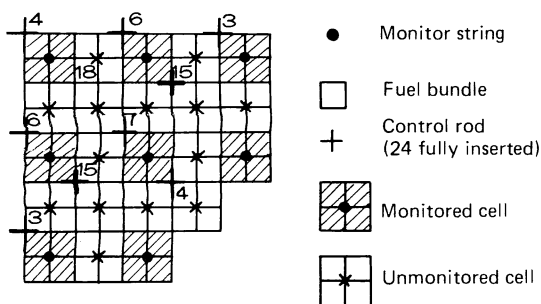


Fig. 1. Monitored and unmonitored cells and an example of a control rod pattern.

<sup>7</sup>J. F. CAREW, "Process Computer Performance Evaluation Accuracy," NEDO-20340, General Electric Company (1974).

<sup>8</sup>Y. NISHIZAWA, T. KIGUCHI, and H. MOTODA, *Nucl. Sci. Eng.*, **60**, 189 (1976).

<sup>9</sup>D. L. DELP et al., "FLARE: A Three-Dimensional Boiling Water Reactor Simulator," GEAP-4598, General Electric Company (1964).

In the present method, calculation of power distributions is based on a FLARE-type nodal coupling method. The nodal coupling method is a practical technique because of its relatively short computing time and small memory requirement. Agreement with operating data or results of detailed design calculations is fairly good provided that parameters included in the model, such as mixing kernels, are optimally adjusted.<sup>10</sup>

Figure 2 outlines the procedure for estimating three-dimensional power distributions from detector readings.

### II.A. A Scheme for Calculating Power Distributions in a Monitored Cell

The basic equation of the nodal coupling method for a monitored cell is

$$S_i(k) = \frac{\kappa_{\infty i}(k)}{\lambda} \left\{ W_i^V(k+1) S_i(k+1) + W_i^V(k-1) \right. \\ \times S_i(k-1) + \sum_{j \neq i}^2 W_j^H(k) S_j(k) + [1 - 2W_i^V(k) \\ \left. - 4W_i^H(k) + \alpha_i(k)W_i^H(k)] S_i(k) \right\} \quad (1)$$

Input—operational data, detector readings, nuclear constants, etc.

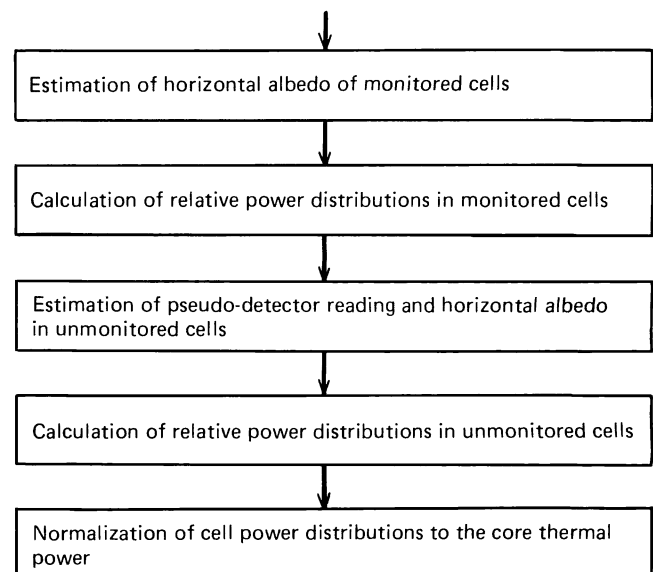


Fig. 2. Procedure for estimating three-dimensional power distributions from detector readings.

<sup>10</sup>T. KIGUCHI and T. KAWAI, *Nucl. Technol.*, **27**, 315 (1975).

where

- $i$  = index of a fuel bundle in a monitored cell
- $j$  = index of two fuel bundles, inside the monitored cell, facing fuel bundle  $i$
- $k$  = axial node number
- $S$  = neutron source
- $\kappa_\infty$  = infinite neutron multiplication factor
- $\lambda$  = eigenvalue
- $W^V$  = vertical neutron transport kernel
- $W^H$  = horizontal neutron transport kernel
- $\alpha$  = horizontal albedo (2.0 for flat boundary condition).

The neutron source  $S_i(k)$  can be obtained from Eq. (1), if the albedo  $\alpha_i(k)$  is given. In this method, the albedos are initially estimated by a simple two-dimensional model, described in Sec. II.B, and are iteratively corrected so that the calculated detector readings are consistent with the measured readings. At each iteration step, the albedo is revised by using the relationship,

$$\alpha_i(k_n) \rightarrow [1 + \epsilon \delta(k_n)] \alpha_i(k_n) \quad , \quad n = 1, 2, \dots, N \quad , \quad (2)$$

where  $\delta(k_n)$  is the correction factor at the axial node  $k_n$ , where the  $n$ 'th detector is located, and  $\epsilon$  is the over-estimation relaxation factor. The quantity  $N$  represents the total number of detectors per monitor string. The factor  $\delta(k_n)$  is calculated by

$$\delta(k_n) = \frac{\Phi_m(k_n) - \Phi_c(k_n)}{\Phi_m(k_n)} \quad . \quad (3)$$

where  $\Phi_m(k_n)$  and  $\Phi_c(k_n)$  are the measured and the calculated detector readings, respectively. The  $\Phi_c(k_n)$  is obtained by

$$\Phi_c(k_n) = f[S_1(k_n), S_2(k_n), S_3(k_n), S_4(k_n)] \quad . \quad (4)$$

Here,  $f$  is a linear function that relates the neutron source of a monitored cell to the detector readings. The correction factors at the other axial nodes with no detector are determined by linear interpolation or extrapolation of  $\delta(k_n)$ . The correction of albedos is repeated until the differences between  $\Phi_c$  and  $\Phi_m$  become sufficiently small. The scheme of albedo correction is essentially the same as that used in Ref. 8, which was successfully applied to one-dimensional problems.

The thermal-hydraulic calculation, coupled with the solution of Eq. (1), requires the thermal power and coolant flow rate of each monitored cell. These values, however, are unknown until the power distribution in the whole core is obtained. Therefore, the present method allows the use of their estimated values to perform the power-void interactions to obtain the power distribution in each cell. These two

variables mainly affect the degree of axial power skewing, but the error due to their uncertainty can be reduced by the albedo correction.

### II.B. Estimation of Horizontal Albedo

The horizontal albedo,  $\alpha_i(k)$ , for a fuel bundle in a cell is defined by

$$\alpha_i(k) = \frac{\sum_{j=1}^2 W_j^H(k) S_j(k)}{W_i^H(k) S_i(k)} \quad , \quad (5)$$

where the summation is over the two adjacent fuel bundles outside the cell considered. To calculate  $\alpha_i(k)$  by Eq. (5), it is only necessary to know the relative magnitudes of  $S_j(k)$  of neighboring fuel bundles, and not an accurate, whole-core distribution. Therefore, a two-dimensional FLARE-type calculation, neglecting the effect of axial neutron leakage, can be used to get the horizontal source distribution for calculating  $\alpha_i(k)$  values.

In the present method, to improve the accuracy of  $\alpha_i(k)$ , a horizontal distribution of detector readings is used for modification of the results of the two-dimensional calculation at each axial node,  $k_n$ , where the  $n$ 'th detectors are located in the monitored cells. The modification is performed by linear interpolation or extrapolation of the ratios of the actual detector readings to the calculated values obtained from the results of the two-dimensional calculations.

Another modification is considered with respect to the axial change of the albedos: they generally change across the two axial nodes,  $k_n$  and  $k_{n+1}$ , especially if the neighboring control rod pattern changes between the nodes. Such changes are estimated by use of a simple perturbation scheme on the basis of the assumption that the change in neutron source distribution takes place at the tip of the control rods. The basic equations of the two-dimensional FLARE-type model are shown by Eqs. (6) and (7):

$$S_{i,j} = \frac{\sum W_{m,n}^H S_{m,n}}{A_{i,j}} \quad (6)$$

and

$$A_{i,j} = \frac{\lambda}{\kappa_{\infty,i,j}} - 1 + 4 W_{i,j}^H \quad . \quad (7)$$

The neutron source at the node  $(i^*, j^*, k)$  above the tip of the control rod that is inserted up to the axial node  $(k-1)$  is approximated as follows:

$$S_{i^*, j^*}(k) = S_{i^*, j^*}(k-1) + \Delta S_{i^*, j^*}(k) \quad (8)$$

$$\begin{aligned} &= \frac{1}{A_{i^*, j^*}(k)} \sum W_{m,n}^H(k) S_{m,n}(k-1) \\ &\times \frac{\kappa_{\infty, m, n}(k)}{\kappa_{\infty, m, n}(k-1)} \quad . \quad (9) \end{aligned}$$

The change of the neutron source distribution in the neighborhood of the node ( $i^*, j^*$ ) is approximated by Eq. (10):

$$\Delta S_{i,j}(k) = \frac{1}{A_{i,j}(k)} \sum W_{m,n}^H(k) \Delta S_{m,n}(k) \quad , \quad (10)$$

where

$$\Delta S_{i,j}(k) = S_{i,j}(k) - S_{i,j}(k-1) \quad .$$

The  $\Delta S_{i^*,j^*}(k)$  calculated by Eqs. (8) and (9) is used as a fixed source to solve  $\Delta S_{i,j}(k)$  in Eq. (10). In Eqs. (9) and (10), the value of  $\lambda$  included in  $A_{i,j}(k)$  is set to unity, assuming that the reactor is in a critical condition.

Estimation of the horizontal albedo is performed in accordance with the following steps:

1. calculation of  $S_{i,j}(k_n)$  and  $\alpha_{i,j}(k_n)$  at the axial nodes,  $k_n$ , ( $n = 1, 2, \dots, N$ ), where  $N$  is the number of detectors per monitor string
2. calculation of  $S_{i,j}(k)$  at the axial nodes,  $k$ , ( $k_n < k < k_{n+1}$ ), using the above perturbation scheme, which is successively applied from the node  $k_n$
3. calculation of  $\alpha_{i,j}(k)$  from  $S_{i,j}(k)$ .

The characteristics of the above scheme are its short computing time and small memory requirement by adoption of the two-dimensional model and simple perturbation scheme at the axial nodes between  $k_n$  and  $k_{n+1}$ .

### II.C. Estimation of Pseudo-Detector Readings in Unmonitored Cells

The horizontal neutron source distribution in unmonitored cells at each axial detector height is estimated by the two-dimensional equation, Eq. (11), in which the neutron sources of monitored cells are used as the fixed sources:

$$S_i^{(n)}(k_n) = \frac{\kappa_{\infty}^i(k_n)}{\lambda} \left\{ \sum_{j \neq i} S_j^{(m)}(k_n) W_j^H(k_n) + \sum_{j \neq i} S_j^{(m)}(k_n) W_j^H(k_n) + [1 - 4 W_i^H(k_n) - 2 W_i^V(k_n) + \beta_i(k_n) W_i^V(k_n)] S_i^{(n)}(k_n) \right\} \quad , \quad (11)$$

where  $\beta_i(k_n)$  is a vertical albedo and the superscripts ( $m$ ) and ( $n$ ) indicate monitored and unmonitored cells, respectively. The neutron sources  $S_j^{(m)}(k_n)$  are given by a series of local core calculations described in Sec. II.A.

The vertical albedo, defined in Eq. (12), is necessary to get the solution of Eq. (11).

$$\beta_i(k_n) = \frac{S_i^{(n)}(k_n + 1) W_i^V(k_n + 1) + S_i^{(n)}(k_n - 1) W_i^V(k_n - 1)}{S_i^{(n)}(k_n) W_i^V(k_n)} \quad . \quad (12)$$

Since a control rod is inserted at the corner of one monitored and three unmonitored cells, the axial distribution in a fuel bundle of an unmonitored cell is considered to be similar to that in the nearest fuel bundle of a monitored cell. Therefore, the vertical albedo,  $\beta_i(k)$ , of an unmonitored cell can be approximated by Eq. (12), where  $S_i^{(m)}$  of the monitored cell, nearest to the bundle  $i$ , is substituted for  $S_i^{(n)}$ .

The neutron source distribution in monitored cells used as the fixed source in Eq. (11) is consistent with the measured detector readings, and the detector readings obtained from  $S_i^{(n)}$  in Eq. (11), by Eq. (4), can be expected to be good estimates of the pseudo-detector readings in unmonitored cells. The power distribution in unmonitored cells can be estimated, using the pseudo-detector readings, by the same scheme as in the case of monitored cells.

### II.D. Normalization of the Cell Power Distribution to the Core Thermal Power

To normalize the power distributions in the cells to the core thermal power, a normalization factor is defined for each cell [as in Eq. (13)] as

$$a_j = \frac{1}{N} \sum_{n=1}^N \{ \Phi^j(k_n) / f [s_1^j(k_n), s_2^j(k_n), s_3^j(k_n), s_4^j(k_n)] \} \quad , \quad (13)$$

where

$a_j$  = normalization factor for cell  $j$

$\Phi^j$  = measured detector readings in the case of monitored cells, or pseudo-detector readings in the case of unmonitored cells

$s_i^j$  = neutron source distribution normalized to the average value of 1.0 in cell  $j$

$f$  = a linear function that relates the neutron source of the cell to the detector readings, the same as Eq. (4).

When the summation of the  $a_j$  values is normalized to the total core thermal power,  $a_j$  is equal to the thermal power of the cell  $j$ . The power distribution,  $p_i^j(k)$ , normalized to the total core thermal power is obtained by multiplying the source distribution  $s_i^j(k)$  by the above factor, i.e.,

$$p_i^j(k) = s_i^j(k) \cdot a_j \quad . \quad (14)$$

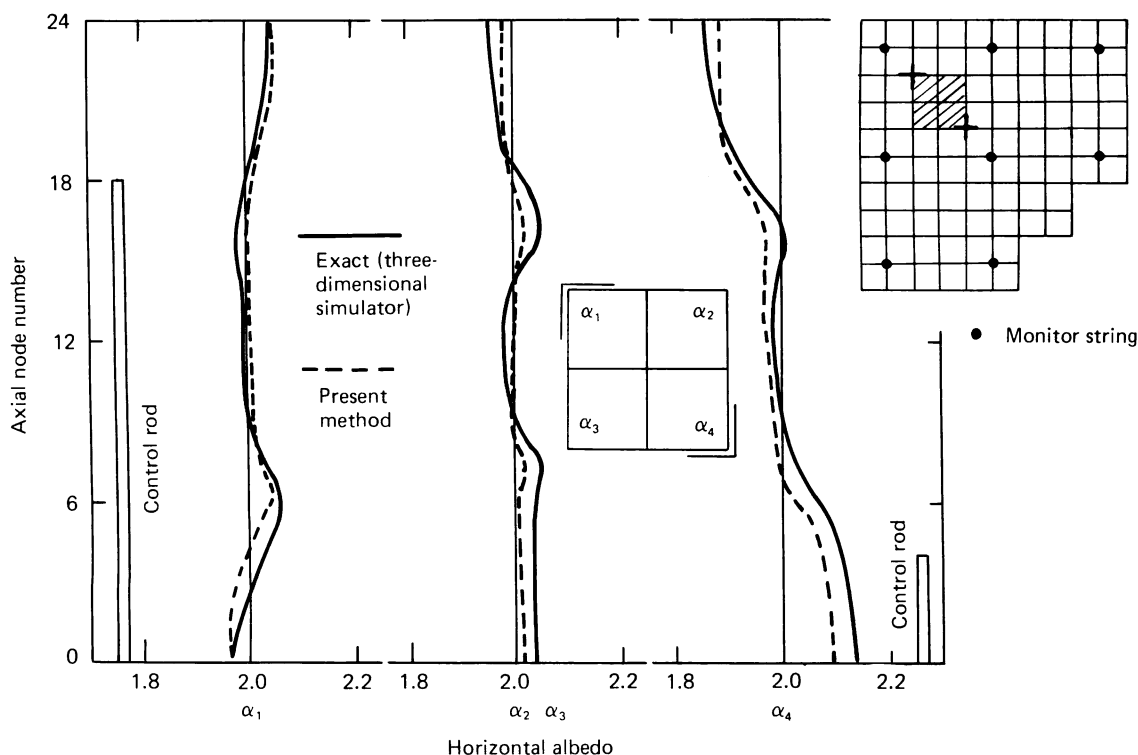


Fig. 3. Axial distributions of horizontal albedo in an unmonitored cell. The shaded area of the insert indicates the cell under consideration.

### III. TEST RESULTS AND DISCUSSION

Numerical tests were performed for the quarter-core model of a BWR shown in Fig. 1. A total of 352 fuel bundles are loaded in the full core, and its rated thermal output and core flow are 1380 MW and  $60.5 \times 10^2$  kg/s ( $48.0 \times 10^6$  lb/h), respectively. There are eight monitored cells in the quarter core. The accuracy of the present method was evaluated by comparison with a reference, exact solution of the three-dimensional whole-core FLARE model.

#### III.A. Case with Four Axial Detectors

First, the case was tested in which each string had four detectors at equally spaced axial locations. This is the standard case for present BWRs.

Figure 3 shows an example of the axial distribution of horizontal albedo obtained by the method described in Sec. II.B, together with those obtained from an accurate solution with the three-dimensional whole-core calculation. The estimated albedos agree

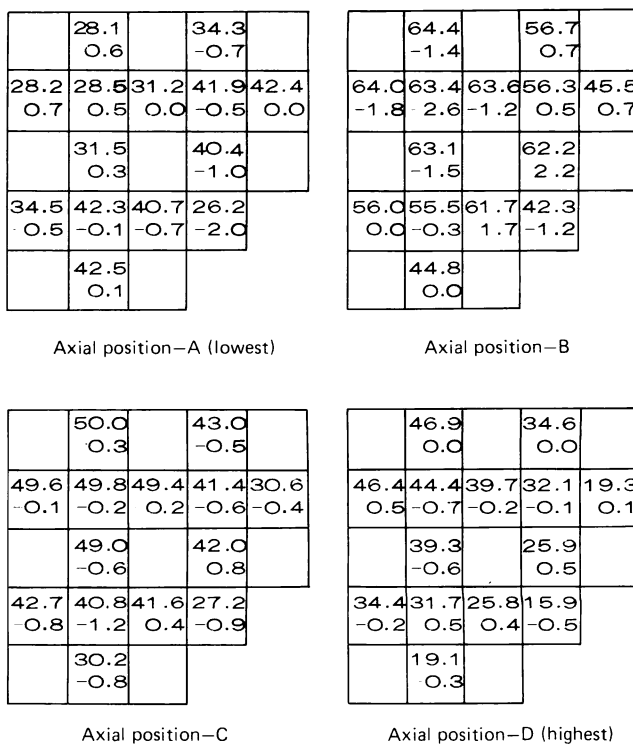


Fig. 4. Estimated detector readings in unmonitored cells and differences from those obtained by three-dimensional whole-core calculation.

XX.X Estimated detector reading  
X.X Difference from exact reading

fairly well with those calculated by the three-dimensional whole-core model. Such agreement enables the use of the simple scheme for the albedo correction with Eqs. (2) and (3), in which the correction is made only for the axial direction, preserving the relation among the values of the four fuel bundles at each axial height.

Figure 4 shows the pseudo-detector readings at four axial heights, estimated by the present method. The discrepancy between the estimated and the reference readings is  $<7\%$ . Figures 5 and 6 show the axial power distributions in monitored and unmonitored cells, respectively.

The normalized distributions of bundle power are given in Fig. 7, and the differences from the exact values in Fig. 8. Generally, the accuracy for monitored cells is higher than that for unmonitored cells. The maximum error of bundle power is 4.5%. For this case, the standard deviation of nodal power distributions in 22 cells is 2.6%.

The computing time for the present case is summarized in Table I. The computing time for one cell is  $\sim 1$  s using an IBM 370/158. The required memory size is also small enough for on-line use.

In general the convergence in the local core model is faster than that in the whole-core model, especially for a large core size. Furthermore, it should be noted here that a simple acceleration scheme using detector readings is efficient for reducing the number of power-void iterations in solving Eq. (1) of the local core FLARE model. Source distribution can be modified in the next iteration by multiplying acceleration factors, which are calculated in the same way as the albedo correction factors in Eq. (3). The efficiency of the acceleration scheme is illustrated by Fig. 9. As shown in the figure, the acceleration is efficient in the early stages of power-void iteration process, and the number of iterations can be reduced to less than half of those without acceleration.

### III.B. Effects of the Number of Detectors

In the operation of present BWRs, there are two important examples in which more information from detector readings can be used than in the previous case. The first is when data from axially movable, in-core detectors such as Traversing In-Core Probes (TIPs) can be used, and the second is when the

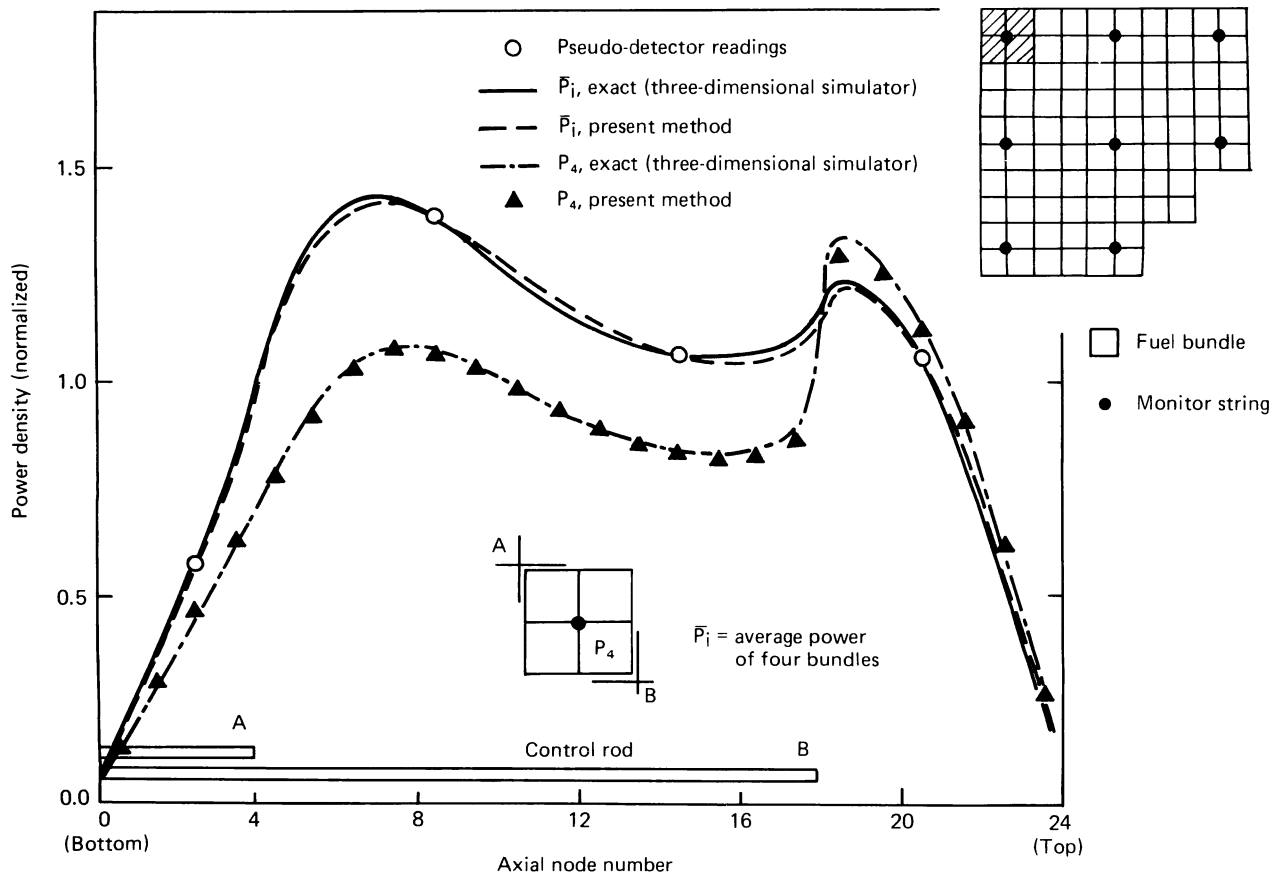


Fig. 5. Power distributions estimated from detector readings in a monitored cell. The shaded area of the insert indicates the cell under consideration.

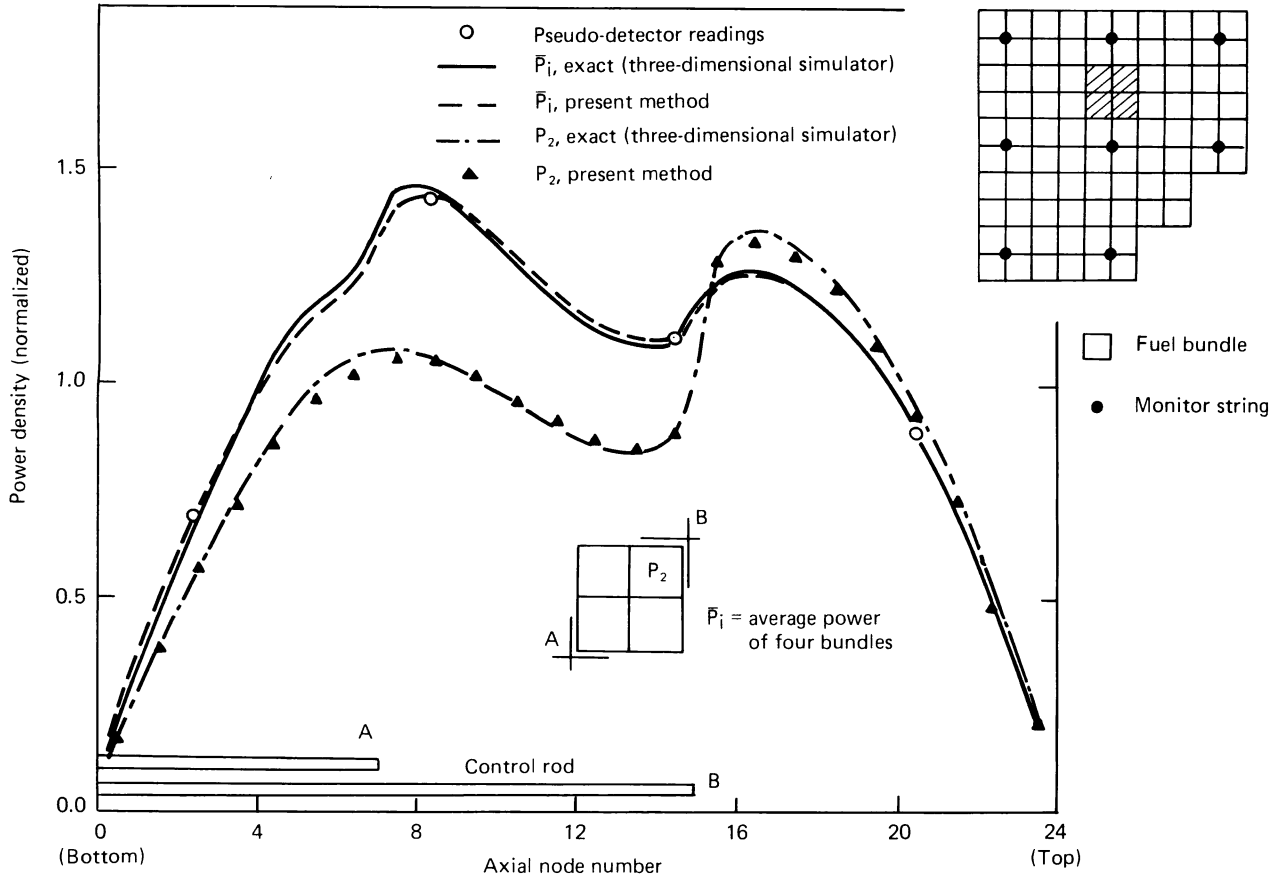


Fig. 6. Power distributions estimated from pseudo-detector readings in an unmonitored cell. The shaded area of the insert indicates the cell under consideration.

reactor is in operation using a quarter-core symmetry. In the first example, the detector readings are available at all axial heights in monitored cells. In the second, most cells can be regarded as monitored cells.

Table II shows the accuracy of estimation of the power distribution for various numbers of detectors. In the case of TIP data, the root-mean-square (rms) error of the nodal power distribution is reduced, but the accuracy for bundle power is not improved

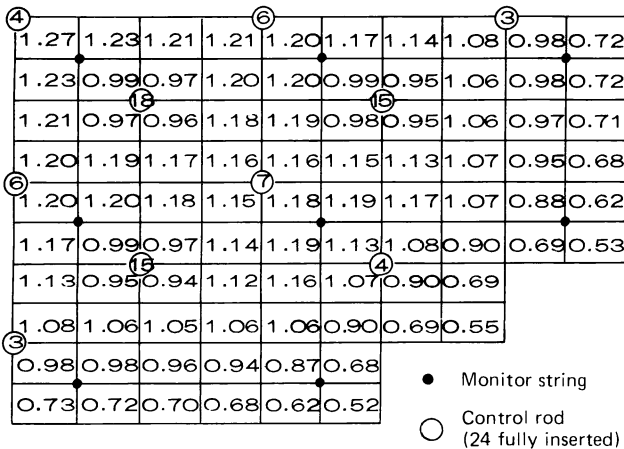


Fig. 7. Normalized bundle power distribution estimated by the present method.

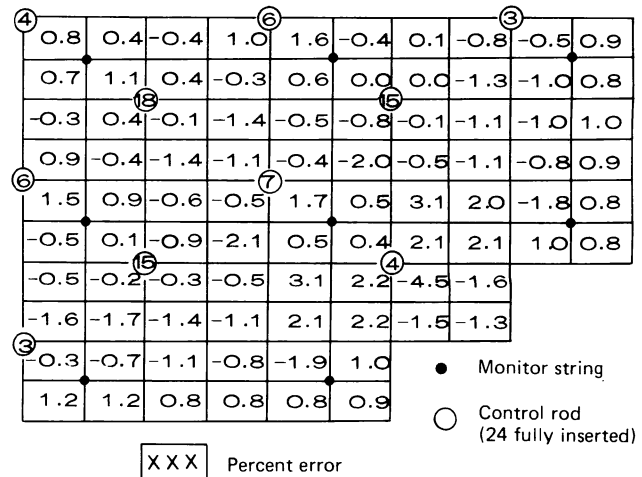


Fig. 8. Accuracy of estimated bundle power distribution.

TABLE I  
Computing Times of Power Distribution Estimation for the Test Calculations

Calculation Step	Dimensions of FLARE Calculation	Number of FLARE Calculation	Computing Time <sup>a</sup> (s)
Estimation of horizontal albedos	Two-dimensional 10 × 10	4 + $n^b$	6
Calculation of power distributions in monitored cells	Three-dimensional 2 × 2 × 24	8	5
Estimation of pseudo-detector readings	Two-dimensional 10 × 10	4	4
Calculation of power distributions in unmonitored cells	Three-dimensional 2 × 2 × 24	14	9
Total			24

<sup>a</sup>Required by an IBM 370/158 computer.

<sup>b</sup>Here,  $n$  is the number of the simple perturbation calculations counted as equivalent to that of the normal eigenvalue calculations ( $n \approx 2$ ).

when compared with the standard case. The accuracy of bundle power depends mainly on the pseudo-detector readings in the unmonitored cells. On the basis of this observation, there is no essential difference between the two cases. However, both the accuracy of nodal power distribution and bundle power are improved for the case of symmetric operation.

### III.C. Effects of Additional Iteration of the Procedure

If the procedure in Fig. 2 is repeated with a minor modification so that, after the first iteration, the horizontal albedos are calculated by Eq. (5) using the results of the previous iteration, the method becomes essentially that of the whole-core FLARE calculation with fixed internal boundary conditions. The values of the thermal power and coolant flow rate of each cell are revised using the results from the power normalization stage in the previous itera-

tion, to perform the power-void iterations in the cell calculations. Accuracy with additional iterations of the procedure is presented in Table III. There is a clear trend of increasing accuracy as the number of iterations is increased, especially in a bundle power. This feature may be useful for on-line use, since it is possible to calculate the power distribution with increasing accuracy, and without two-dimensional whole-core calculations for estimation of horizontal albedo during operation in which the control rod patterns remain almost unchanged.

### IV. CONCLUSIONS

A method has been proposed for estimating three-dimensional power distributions in a BWR core by using in-core detector readings. The power distributions are obtained from a series of three-dimensional nuclear, thermal, and hydraulic calculations for a number of cells, each of which consists of four fuel

TABLE II  
Accuracy of Estimation with Various Numbers of Detectors

	Number of Monitored Cells	Number of Detectors per String	rms Error of Nodal Power Distribution (%)	Maximum Error of Bundle Power (%)
Standard case	8	4	2.6	4.5
With TIP data	8	24	2.0	4.6
With symmetric operation	22	4	2.1	2.0



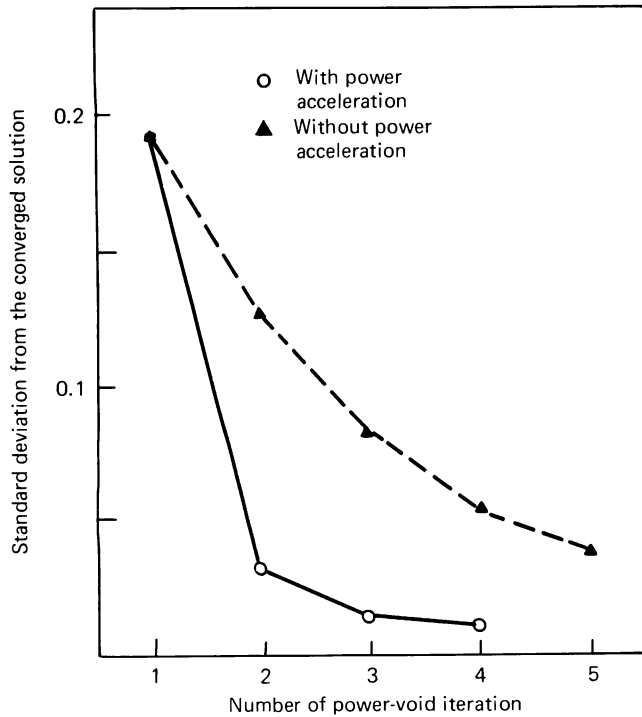


Fig. 9. Convergence in the local core model with or without power acceleration.

bundles surrounding a fixed monitor string, along which several neutron detectors are mounted. The detector readings are used as forced internal boundary conditions in the calculation of cell-power distributions. These are also used in the two-dimensional whole-core calculation model to estimate the outer boundary conditions of cells, so that the power distribution in a cell is consistent with that obtained from the three-dimensional whole-core calculation.

TABLE III  
Accuracy of Estimation with Additional Iterations of the Procedure

Number of Additional Iterations	rms Error of Nodal Power Distribution (%)	Maximum Error of Bundle Power (%)
0	2.6	4.5
2	2.2	1.6
5	1.9	1.0

The method is comprised of the following steps:

1. estimation of the boundary conditions in cells by the two-dimensional nodal coupling method using detector readings
2. calculation of the power distribution in monitored cells, by the three-dimensional nodal coupling method with a model adjusting scheme to make the calculated detector readings consistent with the measured ones
3. estimation of the pseudo-detector readings in unmonitored cells by the two-dimensional fixed-source nodal coupling method
4. calculation of the power distributions in unmonitored cells by the same method as in monitored cells.

The test calculations showed the potential suitability of the proposed method for such on-line computer application as the Core Performance Monitoring System.<sup>11</sup> The method does not require off-line preparation of the fitting constants to convert the detector readings to three-dimensional power distributions. High accuracy is expected under various operating conditions, because the calculation of power distributions is based on a physical model, i.e., a FLARE-type nodal coupling model, and because the model contains parameters, such as boundary conditions, that can be adjusted by using the measured detector readings. The synthesis technique, combining local three-dimensional calculations and whole-core two-dimensional calculations, makes the memory size and computing time small enough for on-line use. The proposed method is, in particular, useful for a core with asymmetric fuel loadings and control rod patterns.

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<sup>11</sup>T. KIGUCHI et al., "On-Line Core Performance Evaluation and Operating Guidance System for BWR," *Proc. Int. Conf. Nuclear Power Plant Control and Instrumentation*, Cannes, France, April 24-28, 1978, IAEA-SM-226/30, p. 449, International Atomic Energy Agency (1978).